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Sylwester Bejger, Joanna Bruzda[†]
Nicolaus Copernicus University in Toruń

Detection of Collusion Equilibrium in an Industry with Application of Wavelet Analysis

A b s t r a c t. In the present paper an attempt was made to verify the possibilities of the use of a marker of structural changes of market price variance in the detection of trade collusion between business players. We used the theoretical model of strategic behaviour of trade players with the assumption of exogenous and time-constant cartel quota (market shares), which justifies the application of a marker for business with specific parameters. The paper contains empirical employment of a marker for a sequence of average Lysine price on the USA market in 1990–1996. Wavelet analysis was applied, for the first time in this context, as the econometric method for the detection of structural changes in the variance.

K e y w o r d s: Explicit and tacit collusion, supergame with a fixed structure of market shares, price variance, wavelet analysis

Introduction

In the paper Bejger (2010), on the basis of well-known Lysine[1] cartel (1990–1996 period) a theoretical model of strategic behaviour of industry players was constructed as a standard supergame model with a Cournot type stage game with additional assumption of exogenous and time-constant cartel quota (market shares). For business branches bound by specific parameters, e.g. Lysine manufacturers[2] in the above mentioned period, the model may indicate certain characteristic structural disorders in the variance of the market price, resulting from the likelihood of the development of price war phase, caused by

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[1] Lysine is an α-amino acid that the human body cannot synthesize. Lysine production for animal feed is a major global industry, as Lysine is an important additive to animal feed because it is a limiting amino acid when optimizing the growth of certain animals such as pigs and chickens for the production of meat.

[2] For a detailed description of Lysine conspiracy see Connor (2001).

a player with no intention of maintaining or establishing collusion due to a too low predicted or factual share on the market or a certain market price stiffness in the collusion phase in the periods of market contraction. Present paper includes an attempt of the application of wavelet analysis with the aim of an empirical verification of the correctness of theoretical findings. The verification is made possible thanks to the well known history of strategic behaviour of the players in the business. The following chapter will introduce the summary of the to-date empirical research with the use of variance change marker. Chapter 2 contains the description of research methodology. Chapter 3 contains empirical work (in a case-study flavour). In chapter 4 the results of the study were presented. Chapter 5 summarizes the whole of the research.

1. To-Date Empirical Research with Application of Variance Change Marker

Assuming that the theoretical model accurately describes strategic behaviour in the industry of Lysine manufacturers, the following research hypotheses could be put forward:

- regime changes of the variance of average Lysine prices are possible due to a probability of the occurrence of price war phases,
- in the collusion phase the variance should be lower than in competition phase.

These two specific patterns in variance process could distinguish collusion from competition and are usually named markers of collusion (Harrington, 2005, p. 25).

With the aim of detecting such variance disorders a wavelet analysis was applied. So far, such approach has not been used. Basing on theoretical findings, one could put forward a hypothesis that, on average, in the collusion phase price variance is lower than in the competition phase. One should also expect regime variance change while switching from collusion phase to competition phase (price war). To-date papers connected with the detection of collusion on the basis of the detection of structural changes in the variance embraced the application of descriptive statistics methods in the comparison of variance levels in the phases of collusion and competition (Abrantes-Metz, Froeb, Geweke, Taylor, 2006), the application of ARCH / GARCH specification in the process of market price including the additional 0-1 variable describing the phases of collusion and competition (Bolotova, Connor, Miller, 2008), as well as the application of Markov switch model of MS(M)(AR(p))GARCH(p,q) type for the variance and/or average (constant) of the price process (Bejger, 2009). The two latter articles are all the more interesting due to the fact that they refer to the cartel of Lysine manufacturers. In the paper (Abrantes-Metz, Froeb, Geweke, Taylor, 2006) the cartel of frozen fish suppliers was the research subject. The existence and functioning of the cartel was confirmed by the authorities.

In the course of the research it was found that during collusion phase (before the discovery of the cartel) market price variance (offered by colluders at bargains) was significantly lower than after the fall of the cartel. The hypothesis that justified the research was the supergame model with SPPE equilibrium. In the article (Bolotova, Connor, Miller, 2008) ARCH / GARCH specification was used with the aim of examining the disturbances in the variance and the average of the Lysine market price process during the phases of competition and collusion. The influence was described through 0-1 variables for the phases of cartel and competition. A statistically significantly lower average variance was reached in the periods of collusion than in the phases of competition. However, it should be underlined that the marking off of phases of both types (with the aim of 0-1variable specification) took place on the basis of a posteriori evidence from the proceedings in the case of the cartel. In the same article a cartel of citric acid producers was studied, with no particular findings as to the lower variance in the collusion phase. The theoretical basis for the marker was SPPE-type equilibrium. In the paper (Bejger, 2009) Markov switching model was employed with the aim of detecting moments of change in the level of variance and the average of Lysine price process. The results confirmed the existence of two variance regimes and a high likelihood of the process staying within the lower variance regime, especially during the second collusion phase. The study seems of importance in view of the absence of assumptions as to the moments of switch. The detection of those moments partly confirmed the history of cartel activity known from the trial evidence.

2. Wavelet Analysis Methodology

Assuming that the theoretical model accurately describes strategic behaviour in the business of Lysine manufacturers, one could put forward the following research hypotheses:

- regime changes of the variance of average Lysine prices are possible due to different phases of price war,
- in the collusion phase the variance should be lower than in the competition phase.

With the aim of detecting such variance disorders a wavelet analysis was applied. So far, such approach has not been used. Comparing it to variance marker-based econometric methods of collusion detection applied before (mentioned in chapter 1), it can be noticed that the wavelet analysis makes it possible to utilize simple methods of statistical inference, allows the preliminary, graphic, estimation of the changes in variance, and as a non-parametric method is not burdened with model specification error. In addition, it allows for the indication of the scales in case of which a change occurs; this however requires the access to long time sequences.

Wavelet analysis consists in the decomposition of the process into components constituting shifted and rescaled versions of the so-called mother wavelet, $\psi(\cdot)$, which is a function with unit energy, fulfilling the so-called admissibility condition (Percival, Walden, 2000, p. 4). Let a vector be given in the form of $\mathbf{x} = (x_0, x_1, \ldots, x_{N-1})$ where $N = 2^J$. For $j = 1, 2, \ldots, J$ and $t = 0, 1, \ldots, 2^{J-j} - 1$ (j – decomposition level, t – wavelet coefficient number) we define discrete wavelet transformation (DWT) of the vector **x**:

$$W_{j,t} = \sum_{n=0}^{N-1} x_n \psi_{j,t}(n), \tag{1}$$

where $\psi_{j,t}(\cdot)$ are versions of the basis wavelet shifted by an integer number and rescaled on the dyadic scale $\lambda_j = 2^{j-1}$, $j = 1,2,\ldots$, i.e.:

$$\psi_{j,t}(x) = 2^{-j/2} \psi\left(2^{-j} x - t\right). \tag{2}$$

In the case of the popular Daubechies wavelets for a given j wavelet coefficients $W_{j,t}$ are proportional to differences (of various orders) of weighted averages over the scale λ_j. For the stochastic process X_t a time-varying wavelet variance is defined in the following way:

$$\sigma_t^2(\lambda_j) = \frac{1}{2\lambda_j} \mathrm{Var}(W_{j,t}). \tag{3}$$

Under the assumption that the quantity above does not depend directly on time [3], we obtain a decomposition of the variance according to the scale in the form (Percival, Walden, 2000, pp. 296-298):

$$\mathrm{Var}(X_t) = \frac{1}{2} \sum_{j=1}^{\infty} \frac{1}{\lambda_j} \mathrm{Var}(W_{j,t}) = \sum_{j=1}^{\infty} \sigma^2(\lambda_j). \tag{4}$$

Wavelet variance on the level j corresponding to the scale $\lambda_j = 2^{j-1}$, $\sigma^2(\lambda_j)$, informs about the changeability of fluctuation in cycles contained approximately in the bracket $2^j - 2^{j+1}$.

In the estimation of wavelet variance and wavelet correlation in practice, DWT is replaced by its modification in the form of MODWT (*maximal overlap discrete wavelet transform*)[4], which does not require handling long ranges being

[3] The assumption is fulfilled also for non-stationary processes on condition that the processes are integrated of order d, while the applied wavelet filter is sufficient to eliminate the nonstationarity, i.e. it is a Daubechies filter (daublet, symlet or coiflet) of an appropriate length (see Percival, Walden, 2000).

[4] MODWT is preferably pronounced ‘mod WT’ – modified WT see: (Percival, Walden, 2000, p. 159). Other names for this kind of transformation are *non-decimated wavelet transformation*, *continuous-discrete wavelet transformation* or French-derived term *algorithme à trous*.

the subsequent powers of the number 2, provides a more effective estimator $\sigma^2(\lambda_j)$ and has invariance properties due to shifts in time. (Percival, Walden, 2000, pp. 308-310; Gençay and others, 2002, p. 135). The estimator of wavelet variance is expressed by means on the following formula:

$$\widetilde{\sigma}^2(\lambda_j) = \frac{1}{\widetilde{N}_j} \sum_{t=L_j-1}^{N-1} \widetilde{W}_{j,t}^2 , \tag{5}$$

where $\widetilde{W}_{j,t}$ are MODWT coefficients, $L_j = (2^j - 1)(L-1) + 1$ is the length of the wavelet filter for the scale λ_j (L is the length of the basic wavelet filter), whereas $\widetilde{N}_j = N - L_j + 1$ is the number of coefficients distorted by the extrapolation at the ends of the sample. (1−α)-per cent confidence interval for $\sigma^2(\lambda_j)$ can be approximated in the following way:

$$\widetilde{\sigma}^2(\lambda_j) \pm \varsigma_{\frac{\alpha}{2}} \left(\frac{\hat{f}_{\widetilde{W},j}(0)}{\widetilde{N}_j} \right)^{0,5} , \tag{6}$$

where $\varsigma_{\frac{\alpha}{2}}$ is the (1−α/2) quantile of the normal distribution, whereas $\hat{f}_{\widetilde{W},j}(0)$ is an estimate of the spectral density function for squares of wavelet coefficients for scale λ_j at 0.

2.1. Testing Homogeneity of Variance

The attractiveness of wavelet coefficients in testing volatility changes results from their two vital properties. Firstly, wavelet coefficients are closely tied to the changes at various scales and moments in time, and thus they carry information about the variability of the process. Secondly, (conventional) wavelet transformation provides coefficients which can be treated as approximately non-correlated both in the case of short and long-memory processes, which significantly simplifies the inference procedures (comp. Whitcher, 1998, ch. 4.1.2; Percival, Walden, 2000, p. 351). Meanwhile, in the estimation of the switch moment the employment of MODWT is proposed on the account of eliminating subsampling effects, which entails higher precision of an estimate. The approach put forward in the PhD dissertation of Whitcher (1998) is presented below, and the direct application of the Inclán and Tiao (1994) method to wavelet coefficients is proposed. It is obligatory to stress that the methods of wavelet detection of changes in variance are discussed primarily in the context of long-memory processes, but – because the property of the approximate decorrelation is valid also for the short-memory processes – it is possible to apply the method in the case of the latter processes as well.

Let $\{W_{j,t}\}$ be wavelet coefficients of the jth decomposition level. We are now interested in testing the following hypothesis:

$$\mathrm{H}_0 : \mathrm{Var}(W_{j,L'_j}) = \mathrm{Var}(W_{j,L'_j+1}) = \ldots = \mathrm{Var}(W_{j,N/2^j-1}) ,$$

where L'_j is the number of boundary coefficients of the wavelet transform at the *j*th decomposition level (DWT coefficients, the value of which is influenced by the extrapolation method at the ends of the sample)5. Furthermore, we assume that the wavelet decorrelation is effective what means that the coefficients $\{W_{j,t}\}$ form a second-order Gaussian white noise. Moreover, we assume that the length of the wavelet filter is sufficient for the elimination of deterministic components, i.e. $\mathrm{E}(W_{j,t}) = 0$. The statistics in the test against the alternative hypothesis in the form:

$$\mathrm{H}_1 : \mathrm{Var}(W_{j,L'_j}) = \ldots = \mathrm{Var}(W_{j,k}) \neq \mathrm{Var}(W_{j,k+1}) \ldots = \mathrm{Var}(W_{j,N/2^j-1}) ,$$

where *k* is the unknown location of the variance change, is based on the normalized cumulated sum of squares (CUSUM for squares):

$$W_k = \frac{\sum_{t=L'_j}^{k} W_{j,t}^2}{\sum_{t=L'_j}^{N/2^j-1} W_{j,t}^2} , \tag{7}$$

and have the forms:

A. (Inclán, Tiao, 1994) $$IT = \sqrt{\frac{N'_j}{2}} \max_{k=L'_j,\ldots,2^{J-j}-1} \left| W_k - \frac{k-L'_j+1}{N'_j} \right| ; \tag{8}$$

B. (Whitcher, 1998) $$W = \sqrt{\frac{N'_j}{2}} \max\left\{D^+, D^-\right\}, \tag{9}$$

where:

$$D^+ = \max_{k=L'_j,\ldots,N/2^j-2} \left(\frac{k-L'_j+1}{N'_j-1} - W_k \right), \qquad D^- = \max_{k=L'_j,\ldots,N/2^j-2} \left(W_k - \frac{k-L'_j}{N'_j-1} \right),$$

whereas $N'_j = N/2^j - L'_j$ is the number of coefficients distorted by the extrapolation method at the ends of the sample. In the case where N'_j amounts to at least 128, the following distribution for the test statistics is used (see: Inclán, Tiao 1994, p. 923; Whitcher 1998, p. 60). Small sample critical values (9) can be found in the works of Whitcher (1998), Whitcher and others (2002).

[5] For the applied wavelets those values amount to 0 in the case of the Haar wavelet whereas in the case of d4 wavelet – 1 for the first decomposition level and 2 for higher levels – compare (Percival, Walden, 2000, p. 136).

2.2. Estimating the Location of Variance Change

$\widetilde{k}^*$, for which the appropriate expressions in test statistics reach their maximum As mentioned earlier, it is necessary to base the estimation of the location of variance change on the coefficients $\widetilde{W}_{j,t}$ of the non-decimated wavelet transformation. It is then that the index values, indicates the moment of change. Obviously, in such statistics the lower and upper summation limits must be modified accordingly, in the way so that all non-boundary coefficients are taken into account. And thus, the statistic in Inclán and Tiao version now takes the form:

$$IT = \sqrt{\tfrac{N-L_j+1}{2}} \max_{k=L_j-1,\ldots,N-1} \left| W_k - \tfrac{k-L_j+2}{N-L_j+1} \right|, \tag{10}$$

whereas the expressions in statistics (9) transform into:

$$D^+ = \max_{k=L_j-1,\ldots,N-2} \left(\tfrac{k-L_j+2}{N-L_j} - W_k \right), \quad D^- = \max_{k=L_j-1,\ldots,N-2} \left(W_k - \tfrac{k-L_j+1}{N-L_j} \right),$$

then the normalized cumulated sum of squares is in the form:

$$W_k = \frac{\sum_{t=L_j-1}^{k} \widetilde{W}_{j,t}^2}{\sum_{t=L_j-1}^{N-1} \widetilde{W}_{j,t}^2}. \tag{11}$$

Finally, we receive the moment of change in real time, by applying additional corrections due to a phase shift[6].

In the case in which there is a possibility of the occurrence of multiple changes in variance, it is proposed (Inclán, Tiao, 1994) to apply a procedure described as the algorithm of Iterated Cumulative Sums of Squares, ICSS, based on the method of binary segmentation. The application of such an algorithm in the context of wavelet detection was proposed by Whitcher et al. (2000). The course of the algorithm can be depicted in the following way:

- having a succession of non-boundary DWT coefficients: $W_{j,L'_j}, W_{j,L'_j+1}, \ldots, W_{j,N/2^j-1}$ we assume $t_1 = L'_j, t_2 = N/2^j - 1$ and compute the value of the statistic *IT* or *W* for the range $[t_1, t_2]$. If the statistic points to the occurrence of change, we mark the moment k^*, for which its value was determined and we pass on to the next stage.
- We determine the value of the test statistic to the left of k^*, i.e. in the range $[t_1, k^* - 1]$ and to the right of k^*, i.e. in the interval $[k^* + 1, t_2]$. We keep

[6] In the case of the filters applied further in the empirical part, the evaluation of the moment of change in real time, with the assumption that the time is numbered beginning with 1, is obtained as follows: in the case of the Haar wavelet without any modification of $\widetilde{k}^*$, while in the case of d4 wavelet – by subtracting 1 from $\widetilde{k}^*$.

finding subsequent points by following the same procedure – we appropriately divide the intervals and determine the values of statistics in the smaller ranges of coefficients. The procedure comes to an end when the rejection of the zero hypothesis does not occur.

- We arrange the detected points in the ascending order $k_1, k_2, \ldots, k_n$ while additionally assuming $k_0 = L'_j$ and $k_{n+1} = N/2^j - 1$. For each j = 1, …, n we carry out a test in the interval between the points of change adjacent to k_j, i.e. in the area between k_{j-1} and k_{j+1}. If a potential change-point is not detected again, we reject k_j from the considered set. The procedure is continued on a new set of points and we ultimately finish when there are no reductions in the number of detected changes.

With the aim of selecting the appropriate manner of implementation for the presented methods, it is worth discussing their statistical properties. Firstly, wavelets with smaller support (shorter wavelet filters) have better localization properties – out of the simulation analyses presented in Whitcher's PhD dissertation (1998) we can infer that the localization of the moment of switch is burdened with a bigger mistake for longer wavelet filters. Secondly, wavelet coefficients on the second decomposition level require higher SNR (*signal to noise ratios*), in order to attain the same level of accuracy as on the first level, both in terms of the detection of change and its localization – thus it is vital to make use of primarily first levels of decomposition. Other conclusions of the simulation analyses are as follows: the estimation of the moment of change is lightly biased towards the centre of the test (Inclán, Tiao, 1994; Whitcher, 1998), in the test of homogeneity of the wavelet variance, the Haar, d4 and la8 wavelets provide similar rejection rates in tests for one or two changes in a wide range of value for the parameter d of fractional integration (0,05–0,45), especially if small sample critical values were used and the test was carried out on low (1–2) decomposition levels (Whitcher, 1998; Whitcher and others, 2002). Obviously, Daubechies wavelets (see Daubechies, 1992; Percival, Walden, 2000) with longer support allow to analyse non-stationary processes of a more complicated structure but – simultaneously – render a smaller number of useable coefficients. In the case of the tested sequence we do not analyse higher levels of decomposition also for the reason that it would only contain 19 coefficients (so there is a slim chance of as many as two moments of change being discovered), moreover, wishing to carry out a *j*-level wavelet analysis without having to write in any additional observations we should be equipped with a series of length which is a multiple of 2^j (the length of our series – 78 – does not allow to take the analysis to higher levels without writing in additional observations). Summing up, two wavelets will be applied in the analysis: the Haar wavelet and d4, yielding filters of the length of 2 and 4 respectively[7]. In the testing for homogeneity of variance we will use the first decomposition level.

3. Empirical Study

In the first stage of the analysis graphs were made of the MODWT coefficients and the wavelet variance in its rolling version. Figure 1 was then marked with MODWT coefficients obtained by means of the Haar wavelet on 4 levels of decomposition, together with scaling coefficients from the fourth level as well as the original time line. The values are shifted on the time axis in the way so as to cancel the phase shift.

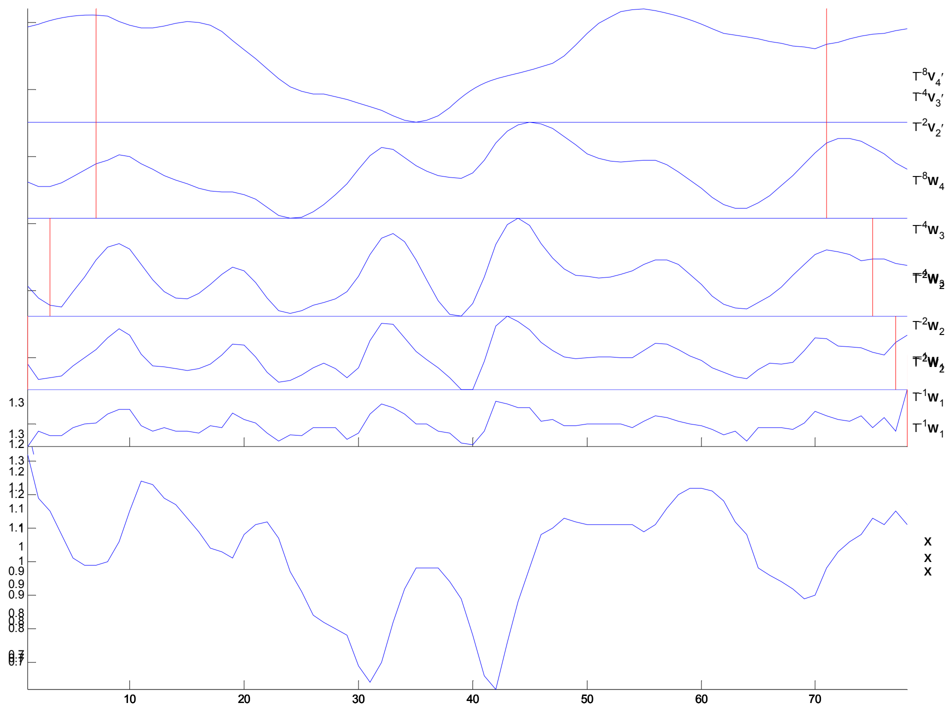

Figure 1. Wavelet, scaling coefficients and the time series. The vertical lines delineate the boundary regions

[7] The d4 wavelet cannot be written in closed form, while the Haar wavelet is given as: $\psi(x) = \mathbf{1}_{<1/2,1)}(x) - \mathbf{1}_{<0,1/2)}(x)$. A detailed presentation of the level *j* Haar and d4 wavelet filters can be found in Percival and Walden (2000), ch. IV.

Figures 2 and 3 (next page) depict the wavelet variance in the rolling (local) version, computed with small portions of wavelet coefficients, shifted on the time axis, together with 95-percent confidence intervals, estimated on the basis of MODWT. The calculations omit all boundary coefficients (i.e. the coefficients the value of which is influence by the extrapolation method at the ends of the series). Two moments can be seen: one located around observation 28 and the other further off.

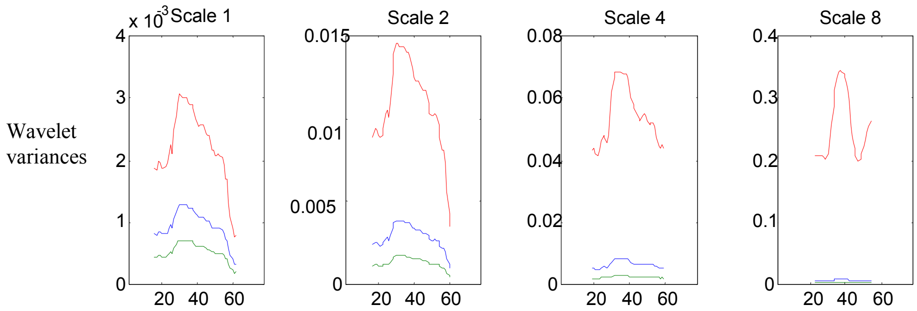

Figure 2. Rolling wavelet variances with with 95-percent confidence intervals – Haar wavelet, window length 30

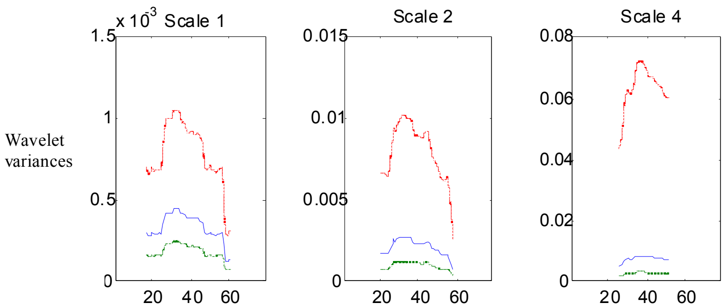

Figure 3. Rolling wavelet variances with with 95-percent confidence intervals – d4 wavelet, window length 30

The next stage consisted in testing of structural changes and variances by means of wavelet methods and included a direct application of the Inclán and Tiao method to DWT coefficients and testing according to Whitcher's wavelet method. The results of both tests are summed up in Tables 1 and 2.

The occurrence of the subsequent point of change was also verified with the application of wavelet d4. This time coefficients with numbers from the interval [1, 20] were evaluated. Appropriate statistics were equal: 0.4106 (IT), 0.6290 (W), pointing to the lack of another switch within the variance. The d4 wavelet indicates two moments of change: we infer about the first one at the low significance level, i.e. empirical significance level remains below 10%, yet the inference about the presence of the latter is already made at 1% level. The observations are made in accordance with the course of rolling wavelet variances.

Table 1. Testing for change in variance – the Haar wavelet

Method and stage of analysis	Statistics	No. of the DWT coefficient (k_j)	Location of the change in variance with the help of the MODWT coefficients
IT - first change-point	1.4144**	22 (k_1)	45
W – first change-point	1.3458**	22 (k_1)	45
IT – second change-point searched to the left from the first	0.4627	10	38
W – second change-point searched to the left from the first	0.7356	18	38
IT – second change-point searched to the right from the first	0.5335	27	61
W – second change-point searched to the right from the first	0.8494	34	61

Note: IT – Inclán and Tiao test; W –Whitcher test; asymptotic critical values: 1.224 (10%), 1.358 (5%), 1.628 (1%), small-sample critical values (Whitcher, 1998): n = 16 – 1.135 (10%), 1.265 (5%), 1.508 (1%), n = 32 – 1.157 (10%), 1.293 (5%), 1.553 (1%); small-sample critical values (Inclán, Tiao, 1994): n = 100 – 1.14 (10%), 1.27 (5%), 1.52 (1%); numeration of the DWT coefficients: 0, 1,…, 38; location of variance change in real time; '**' denotes rejection of the homogeneity of variance at the 5% significance level.

Table 2. Testing for change in variance – the d4 wavelet

Method and stage of analysis	Statistics	No. of the DWT coefficient (k_j)	Location of the change in variance with the help of the MODWT coefficients
IT - first change-point	1.1297	23 (k_2)	42
W – first change-point	1.0583	23	42
IT – second change-point searched to the left from the first	1.2241*	20 (k_1)	28
W – second change-point searched to the left from the first	1.5256***	20 (k_1)	28
IT – second change-point searched to the right from the first	1.0331	34	63
W – second change-point searched to the right from the first	0.6940	34	63

Note: IT – Inclán and Tiao test; W –Whitcher test; asymptotic critical values: 1.224 (10%), 1.358 (5%), 1.628 (1%), small-sample critical values (Whitcher, 1998): n = 16 – 1.135 (10%), 1.265 (5%), 1.508 (1%), n = 32 – 1.157 (10%), 1.293 (5%), 1.553 (1%); small-sample critical values (Inclán, Tiao, 1994): n = 100 – 1.14 (10%), 1.27 (5%), 1.52 (1%); numeration of the DWT coefficients: 0, 1,…, 38; location of variance change in real time; designation of the coefficients indicating a change in variance is in an increasing order; '*', '***' denote rejection of the homogeneity of variance at the 10%, 1% significance level, respectively.

As far as the moments of switch are concerned, certain ambiguities appear. The first change-point is in the interval 42–46 (rather closer to 45–46, as the Haar wavelet has better localization properties, and moreover – as it was already mentioned – the evaluation of the moment of change is burdened with error towards the centre of the sample), while the other one – as it results from the test with the use of d4 wavelet as well as the charts – around observations 28–29.

4. Interpretation of Results

In order to better illustrate the received results, the series of average Lysine prices was presented in Figure 4, together with observation numbers.

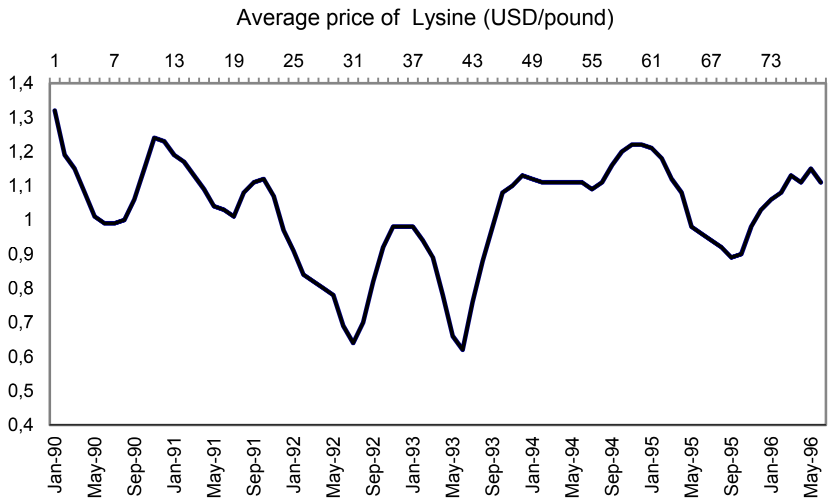

Figure 4. Average price of Lysine, observation numbers on upper horizontal axis. Data from Connor, (2000), appendix A, Table A2

As results from the study we carried out, the moments of variance change are identified around observation 28 – the increase of the variance – and around observation 45 – the fall of the process' variance. This piece of statistical information is confirmed by the analysis of charts 1 and 2. Basing on theoretical motivation of the marker of collusion it is mandatory to make the following statements:

a) until April 1992 no significant change of the variance regime can be found, therefore the period cannot be considered a factual phase of the price war,
b) from April 1992 until August 1993 we can talk about a significant phase of the growth of the process variance, which points to the price war. It is the period of the announcement by ADM (precisely April 1992) of the proposition to establish the 'amino acids manufacturer society', until August 1993, which is around two months before the Irvine meeting (where representatives of two main players, AMD and Ajinomoto had confidentially agreed cartel quotas of Lysine supply, as they further witnessed),

c) from September 1993, therefore exactly a month before the Irvine meeting, a meaningful drop of the process variance occurs, which may indicate the establishment of collusion and the willingness to maintain it. There might be a certain mistake in pinpointing the moment of change or perhaps the players had gone into informal agreements before the time they indicated in their testimonies.

Comparing the results of the wavelet analysis with familiar facts from the cartel history (see: Connor, 2000), it is necessary to notice that the analysis pointed out significant changes in the process variance. The discovered moments of change are closely tied to vital facts from the cartel history, therefore we can assume that they indicate the change in the players' price behaviour. If we accept the proposed game theory model (Bejger, 2010) as the theoretical base for that behaviour, changes in the variance correspond to phases of the price war and collusion. In confrontation with the to-date analyses of the workings of the cartel, the present empirical study leads to the following differences in the history evaluation:

- price war phase is delayed by around 12 months,
- first phase of collusion is not clearly distinguished.

On the one hand, those differences may be caused by a small sample series, which disables wavelet analysis at higher decomposition levels, on the other they may point to a history of Lysine industry collusion that is much different from the one generally accepted.

Conclusions

Taking as an theoretical motivation the model of strategic behaviour of the players in an industry with the assumption of time-fixed cartel quotas (market shares) an attempt of explaining the behaviour of the players in the industry of Lysine manufacturers in the years 1990 – 1996 was made. The model provides theoretical support to the method of the detection of collusion based on the analysis of the variance of the market price process, as well. For empirical evaluation the marker of variance change was applied for a series of average Lysine prices on the USA market in the studied period. Wavelet analysis was employed, for the first time in such a context, as the econometric method of the detection of structural changes in the variance. The proposed method of detection proved to be very useful, indicating the significant changes of the variance regime and precisely detecting the moments of such changes, closely connected to the key dates in the history of the cartel. While enumerating the advantages of wavelet analysis in the proposed application, it is essential to mention:

- parsimony of specification – as a non-parametric method it is not burdened with specification error of the econometric model,

- simplicity of application – the work indispensable to apply the method to the data is minimal. Therefore, the method is quick in application,
- precise indication of the moment of variance change, without any assumptions as to their localization. The method is thus absolutely objective,
- possibility of preliminary, graphic assessment of the variance changes by means of MODWT charts as well as rolling wavelet variances.

Among the drawbacks to wavelet analysis we can indicate relatively high requirements as to the length of the observation series and the lack of the direct link of the method to the structure of equilibrium strategy (as it takes place in e.g. the application of Markov switching model of MS-AR-GARCH type). Therefore, it can be assessed that wavelet analysis in the proposed application may serve as a preliminary detector of the variance changes because its application is cheap and quick with accurate findings. Wherever it is theoretically justified other methods may then be applied with the aim of further verification of the hypothesis of collusion in an industry. Our work has a case –study character so our conclusions should be verified in further research. Next steps in the study will be the modification of the theoretical model for the strategies with different penal codes and – on the empirical plane – an attempt to apply the wavelet analysis of variance for other price time series in industries susceptible to explicit or tacit collusion.

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Detekcja równowagi zmowy w branży z wykorzystaniem analizy falkowej

Z a r y s t r e ś c i. Artykuł zawiera empiryczne zastosowanie markera zmian strukturalnych wariancji procesu ceny rynkowej dla szeregu cen średnich Lysiny na rynku USA w latach 1990–1996. Jako metodę ekonometryczną detekcji zmian strukturalnych w wariancji zastosowano, po raz pierwszy w tym kontekście, analizę falkową. Metoda ta ma w omawianym zakresie aplikacji istotne zalety, takie jak oszczędność w zakresie wymaganych danych statystycznych oraz bardzo dobre własności lokalizacyjne w dziedzinie czasu. W pracy wykorzystano model teoretyczny zachowań strategicznych graczy w branży motywujący zastosowanie wymienionego markera.

S ł o w a k l u c z o w e: Zmowa jawna i milcząca, supergra ze stała strukturą udziałów w rynku, lysina, wariancja ceny, analiza falkowa.