DYNAMIC ECONOMETRIC MODELS
Vol. 9 – Nicolaus Copernicus University – Toruń – 2009

Tomasz Chruściński
Nicolaus Copernicus University in Toruń

The Study of Interdependence Between Capital and Currency Markets Using Multivariate GARCH Models

A b s t r a c t. In the article an attempt was made to investigate the interaction among the various stock exchanges as well as various exchange rates and then to determine the direction of information flow between capital and currency markets. Tools used in this study are Multivariate GARCH models. Presented results developed an earlier study of World Stock Exchange classification. These stock exchanges will be further analysed according to their interaction.

K e y w o r d s: Multivariate GARCH Model, independence analysis, stock exchange, exchange rate.

1. Introduction

The research on the interaction of capital and currency markets in the world has been going on for many years, but this topic has not lost its popularity. Especially in the last year, and when as a result of the rapid withdrawal of investors from the stock exchange, the inflow of speculative capital, and the bankruptcies of numerous well-known companies, discussions about interpenetrating influences of stock exchanges they have on one another were very hot. At the same time, there was considerable interest attracted into fluctuations in the currency market. The aim of this paper is to examine the relationship between stock exchanges of securities of eligible countries and exchange rates associated with them. The article presents a continuation of earlier studies by the author on the classification of stock exchanges in the world, which now will be analyzed in terms of the direction of information flow between the markets. Multivariate GARCH models are applied to determine the influence on volatility.

2. Methodology

With a map-sharing of stock exchanges, resulting from the author's previous studies[1], an attempt was made to determine the direction in which information flows between the dissociated groups, and thus to examine what their spillover effects are. The study used a multivariate model MGARCH, based on daily rates of return with the major indices and exchange rates. Since it would be difficult to examine changes in all indices, representatives of groups have been chosen. Therefore, for use of the specification of the model's equations, logarithms adopted rate of return (1) of the following indices were accepted: Nasdaq, Dow Jones, S & P500, DAX, CAC40, FTSE250, SSE Comp., WIG20, and the exchange rate representing the countries from which the selected indices. These are: EUR/USD, EUR/GBP, EUR/PLN, USD/GBP, USD/CNY, CNY/GBP, CNY/PLN, CNY/USD, GBP/PLN.

The starting point for analysis are the concepts of conditional expected values, conditional variance-covariance matrix and conditional distribution of standardized residuals of the model, presented in the equation:

$$r_t = 100(\ln P_t - \ln P_{t-1}) = \mu_t + \varepsilon_t = \mu_t + \sqrt{h_t} z_t. \tag{1}$$

P_t, P_{t-1} - value of the index (exchange rate) at time t and t-1, μ_t - conditional expected value of rate of return r_t at time t ($\mu_t = \mathrm{E}[r_t \mid I_{t-1}]$), ε_t - residual of the model, h_t - conditional variance of returns at time t ($h_t = \mathrm{var}[r_t \mid I_{t-1}]$), z_t - independent of the residuals of the model with zero mean and unit variance, $I_{t\text{-}1}$ - information available at time t.

More methodological information about the modelling of stochastic processes can be found in the literature (Osińska, 2006; Fiszeder, 2009). The paper focuses on key issues of MGARCH structure model.

Although it has been over 20 years since Engle and Bollerslev (1986) introduced one-equation *ARCH* and *GARCH* models, their attitude to discrete-time variance of rates of return, is still valid and being developed. A natural extension of *GARCH* models for the analysis of financial markets was introduced by Bollerslev (1988) Multivariate GARCH model (MGARCH). In the empirical part of this paper this model was used to describe the mutual influence of stock markets and exchange rates in the world.

The general form of a multivariate model, which is equivalent to one-equation model GARCH (1,1) defined in the work of Bollerslev, Engle, Wooldridge (1988) called VECH-GARCH, is given by equation:

$$vech(\mathbf{H_\tau}) = vech(\mathbf{W}) + \mathbf{A}vech(\varepsilon_{t-1}^T \cdot \varepsilon_{t-1}) + \mathbf{B}vech(\mathbf{H_{t-1}}), \tag{2}$$

[1] Chruściński (2008).

where $vech(\cdot)$ is an operator of symmetric vector.

Several conditions have to be met for an estimation of the model (2) to be properly conducted. A positive definiteness and stationary of matrix $\mathbf{H}_t$ for each time t shall be provided, what requires the positive definiteness of matrix **A** and **B** and is associated with introducing very complicated nonlinear constraints. The primary consequence of the full form of *VECH* equation is the need to estimate a large number of parameters, which even in the two-dimensional model amounts to 21. These problems resulted in little use the model has found in practice. Models with diagonal matrices **A** and **B**, that reduced the number of estimated parameters and eliminated the *transmittance variance effect* by conditioning elements of matrix $\mathbf{H}_t$ from its past values and error products at time t ($\varepsilon_{i,t}\varepsilon_{j,t}$), turned out to be a solution. The general form of the diagonal model *VECH (DVECH)* is:

$$\mathbf{H_t} = \mathbf{W} + \overline{\mathbf{A}} \circ \left(\varepsilon_{t-1}^T \varepsilon_{t-1}\right) + \overline{\mathbf{B}} \circ \mathbf{H_t}, \tag{3}$$

where: $\overline{\mathbf{A}} = ivech(diag(\mathbf{A}))$, $\overline{\mathbf{B}} = ivech(diag(\mathbf{B}))$, intersection $\mathbf{X} \circ \mathbf{Y}$ is the Hadamard product, and $ivech(\cdot)$ is an inverse operator to $vech(\cdot)$.

After reducing matrices **A** and **B** to the diagonal form, we obtain the final form of the model DVECH, which is an extension of GARCH model (1.1) (see Yang, 2001).

A special variant of the VECH model is the BEKK model, which easily solves the problem of non-positive covariance matrix. However, since it is difficult to obtain stationarity of matrix $\mathbf{H}_t$ as well as a small number of estimated parameters (see Piontek, 2006), similarly applicable are diagonal matrices **A** and **B**, to obtain a model DBEKK (2)

$$\mathbf{H_t} = \mathbf{W^T W} + \mathbf{A^T} \varepsilon_{t-1} \varepsilon_{t-1} \mathbf{A} + \mathbf{B^T H_{t-1} B}. \tag{4}$$

Diagonal forms can be used to estimate each equation of models (3) and (4) respectively, allowing to avoid numerous optimization problems of maximum likelihood method (ML) applied to several equations simultaneously. In the empirical example maximisation of the maximum likelihood function defined by the formula (5) was adopted:

$$LLF = -\frac{1}{2}\sum_{t=1}^{T}\left[\ln\left|\mathbf{H_t}\right| + \varepsilon_t^{'} \mathbf{H_t^{-1}} \varepsilon_t\right] \tag{5}$$

3. Empirical Results

The daily rate of return on the main stock indices of the selected countries and the related exchange rates were used for the analysis. Time series were adjusted for comparability by removing rates of return for periods in which even only one variable was unknown. Finally, 1865 observations for the period from

January 2001 to April 2009 were obtained. For such prepared data diagonal VECH models, diagonal BEKK models, and a constant conditional correlation model CCC have been built. The comparison was conducted to assess which of them matches reality the best. After estimating several dozen models, a decision to use the conditional Student's t-distribution was made. This has given better results of the parameters' estimation than a normal distribution.

Table 1. Comparison of estimation results for given models, according to the AIC criterion

Model	AIC value			Optimal model
	DVECH	CCC	DBEKK	
$r_{CNY/GBP}=f(r_{CAC40}, r_{DAX})$; $r_{DAX}=f(r_{S\&P500}, r_{CAC40})$; $r_{S\&P500}=f(r_{DJIA})$	-19.6769	-19.5990	-19.6505	DVECH
$r_{CNY/PLN}=f(r_{DJIA}, r_{CAC40})$; $r_{CAC40}=f(r_{S\&P500}, r_{DAX})$; $r_{DAX}=f(r_{NASDAQ})$	-20.1400	-20.0380	-20.1282	DVECH
$r_{CNY/EUR}=f(r_{DJIA}, r_{WIG20})$; $r_{CAC40}=f(r_{S\&P500}, r_{DAX})$; $r_{WIG20}=f(r_{FTSE250}, r_{DAX})$	-18.8546	-18.7995	-18.8423	DVECH
$r_{GBP/PLN}=f(r_{S\&P500}, r_{DAX})$; $r_{DAX}=f(r_{FTSE250}, r_{CAC40})$; $r_{WIG20}=f(r_{FTSE250}, r_{DAX})$	-18.5694	-18.5593	-18.5418	DVECH
$r_{EUR/PLN}=f(r_{S\&P500}, r_{CAC40})$; $r_{S\&P500}=f(r_{NASDAQ}, r_{DJIA})$; $r_{CAC40}=f(r_{S\&P500}, r_{DAX})$	-20.0290	-20.0298	-19.9953	CCC
$r_{WIG20}=f(r_{NASDAQ}, r_{DAX})$; $r_{DAX}=f(r_{S\&P500}, r_{CAC40})$; $r_{S\&P500}=f(r_{DJIA})$	-18.1299	-18.1305	-18.1000	CCC
$r_{WIG20}=f(r_{S\&P500}, r_{DAX})$; $r_{DAX}=f(r_{S\&P500}, r_{CAC40})$; $r_{S\&P500}=f(r_{DJIA})$	-18.1394	-18.1414	-18.1102	CCC
$r_{WIG20}=f(r_{DJIA}, r_{DAX})$; $r_{DAX}=f(r_{S\&P500}, r_{CAC40})$; $r_{DJIA}=f(r_{S\&P500})$	-18.1705	-18.1764	-18.1469	CCC
$r_{WIG20}=f(r_{NASDAQ}, r_{FTSE250})$; $r_{FTSE250}=f(r_{S\&P500}, r_{CAC40})$; $r_{CAC40}=f(r_{S\&P500}, r_{DAX})$	-19.0869	-19.0830	-19.0638	DVECH
$r_{WIG20}=f(r_{S\&P500}, r_{FTSE250})$; $r_{FTSE250}=f(r_{S\&P500}, r_{CAC40})$; $r_{S\&P500}=f(r_{DJIA})$	-18.8132	-18.8329	-18.7793	CCC
$r_{WIG20}=f(r_{DJIA}, r_{FTSE250})$; $r_{FTSE250}=f(r_{S\&P500}, r_{CAC40})$; $r_{DJIA}=f(r_{S\&P500})$	-18.8622	-18.8839	-18.8324	CCC
$r_{SSE}=f(r_{FTSE250})$; $r_{FTSE250}=f(r_{S\&P500})$; $r_{S\&P500}=f(r_{NASDAQ}, r_{DJIA})$	-18.5000	-18.5103	-18.4458	CCC
$r_{SSE}=f(r_{FTSE250})$; $r_{FTSE250}=f(r_{S\&P500}, r_{CAC40})$; $r_{WIG20}=f(r_{S\&P500}, r_{DAX})$	-17.7720	-17.7780	-17.7281	CCC

The purpose of this research was to determine the direction of the impact of a stock exchange representing one group on a representative of another stock exchange, the mutual influence of exchange rates on one another and, consequently, to examine the relationship between stock market and currency market. Having available rates of return of 8 indices and 9 exchange rates, hundreds of multi-dimensional combinations have been analysed. Because the publication has limited size only examples of the results of estimated models are presented. The value of AIC has been chosen as the selection criterion. The results of the comparisons are shown in Table 1.

It is clearly visible while analysing the results of the estimation presented in Table 1 that the best models to depict the interaction between stock indices and exchange rates are the diagonal VECH model and the constant conditional correlation model CCC. The same structure of the optimal models occurs for all of the interdependence test results. An exemplary multivariate empirical DVECH model is shown below:

$$r_{EUR/PLN(t)} = -0.000332 - 0.042826 \cdot r_{S\&P500(t-1)} + 0.024953 \cdot r_{CAC40(t-1)} + \varepsilon_t$$

$$r_{S\&P500(t)} = 0.000576 + 0.016579 \cdot r_{NASDAQ(t-1)} - 0.093922 \cdot r_{DJIA(t-1)} + \varepsilon_t$$

$$r_{CAC40(t)} = 0.0007 + 0.420494 \cdot r_{S\&P500(t-1)} - 0.20429 \cdot r_{DAX(t-1)} + \varepsilon_t$$

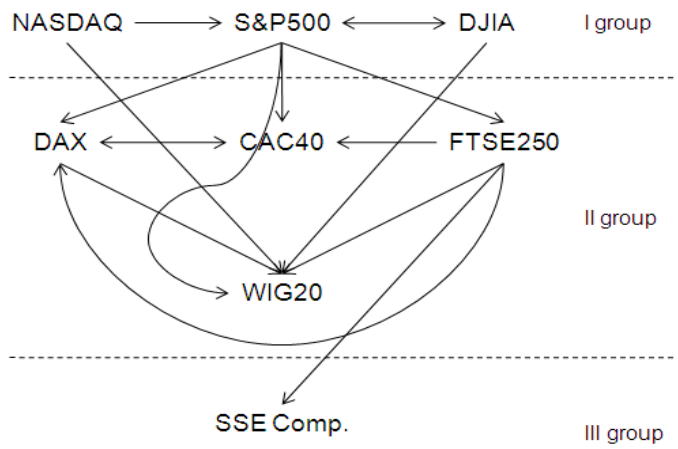

Figure 1. The reciprocal influence of stock exchanges, represented by the major indexes in the period from January 2001 to April 2009

The more interesting part of the calculations' results is an economic interpretation of the constructed multivariate models. Basing on correctly evaluated exogenous variables of individual equations the direction of the information flow from the day before between the individual stock exchanges is clearly visible. An illustration of the mutual influence of stock exchanges used in the study is shown in Figure 1, while Figure 2 represents the results of studies using similar variables from the period from January 2001 to September 2008. A comparison of the two mentioned figures provides us with a clear picture of changes in a correlation between stock exchanges over a 7-month period of the economic crisis.

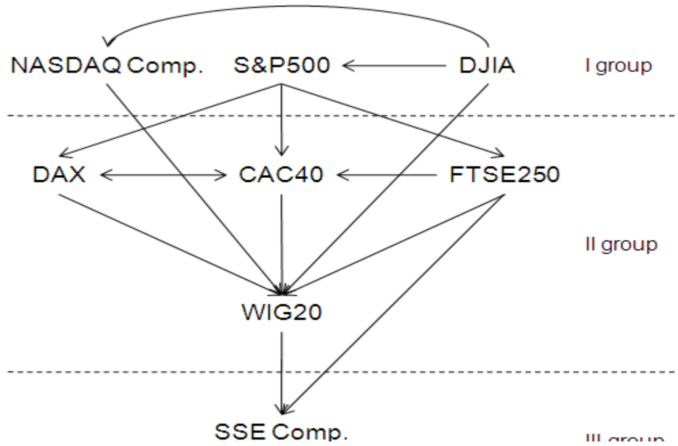

Figure 2. The reciprocal influence of stock exchanges, represented by the major indexes in the period from January 2001 to September 2008

Comparing the results of the analysis shown in Figures 1 and 2, there is a noticeable change in the direction of information flow between the stock exchanges studied, which occurred during the period from October 2008 to April 2009. There is no longer a direct connection between DJIA and NASDAQ indices on the U.S. market, however the London FTSE250 index gained a more direct influence on German DAX index on the European market. Both stock exchanges (British and German) have had an impact on Polish WIG20 index. French stock floor which had previously provided the Polish market with information contained in the S&P500 no longer fulfils this purpose and currently "yesterday's" changes of the American S&P500 directly affect the WIG20. The Warsaw Stock Exchange has lost its position, previously shared with the London Stock Exchange, and does not shape exchange rates in Shanghai anymore.

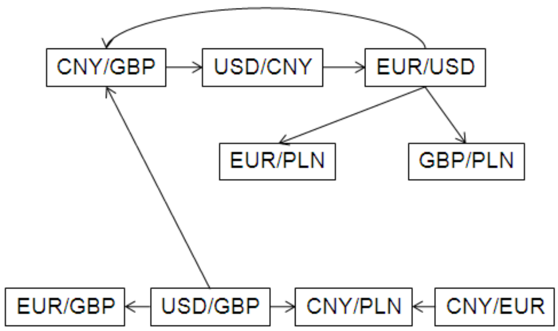

Figure 3. Mutual effect of exchange rates associated with the countries where the analysed stock exchanges come from

The knowledge of dependencies inside the markets is desirable to examine the mutual influence of stock markets and exchange rates. In this case, similarly as for stock exchanges connections, dependencies of rates of return on exchange rates associated with the countries where the investigated stock exchange come from were determined. These dependencies for individual exchange rates are presented in Figure 3.

The analysis allowed to determine the direction of the influence the selected stock exchanges and exchange rates have on one another. The results of these interactions are shown in Figure 4.

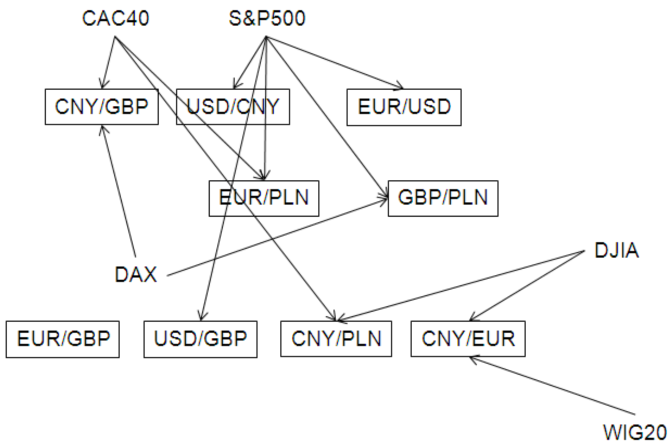

Figure 4. Mutual effect of stock indexes and exchange rates associated with the countries where the analysed stock exchanges come from

4. Summary

The study confirmed the intuitive assumption about the interdependence of stock markets and exchange rates. In majority of cases, exchange rates depend on the stock exchanges indices changes, but not vice versa, what derives directly from investors' tendency to generate profit. In the situation where stock floor is dominated by supply and shares prices of companies are dropping, players withdraw their capital, exacerbating the already declining trend, and invest in a foreign currency. Therefore, the usual changes of stock indices and exchange rates are negatively correlated with one another.

The S&P500 and the CAC40 indices have the biggest influence on exchange rates in a selected group of stock exchanges. Their combined impact shapes, inter alia, EUR/PLN exchange rate. The CNY/PLN exchange rate depends in turn on the DJIA and the CAC40 index. This is due to the fact that the U.S. economy cooperates closely with the Chinese economy and in consequence has a strong influence on the Yuan. From the perspective of Polish stock market it is interesting that the WIG20 index has an impact on the CNY/USD exchange rate. This dependence may result from a shared impact of the non-distant, considering time, Warsaw stock exchange and the London stock exchange on indices of the Shanghai stock exchange. In such a case the connection with SSE exchange is very normal.

In this study, only EUR/GBP exchange rate turned out to be independent of any stock exchange. On the other hand, its value depends on USD/GBP exchange rate. As far as the interdependence of capital markets and exchange rates is concerned, the analytical results presented in the paper are fragmentary. Models computation is a complex and time-consuming process, therefore models comparisons and analysis presented in this paper have an illustrative value only and provide an example of MGARCH models usage.

References

Bollerslev, T., Engle, R., Wooldridge, J. (1988), A Capital Asset Pricing Model with Time-Varying Covariance, *Journal of Political Economy*, University of Chicago Press, 96(1).

Bollerslev, T. (1986), Generalized Autoregressive Conditional Heteroskedasticity, *Journal of Econometrics*, 31, 307–327.

Chruściński, T. (2008), Analiza wielowymiarowa giełd papierów wartościowych na świecie (Multivariate Analysis of Stock Exchanges in the World), *Wiadomości statystyczne* (*Statistical News*), GUS, Warszawa, No. 9, 50–61.

Chruściński, T. (2009), Wykorzystanie wielowymiarowych modeli klasy GARCH do badania wzajemnego wpływu rynków finansowych na świecie (Using multivariate GARCH Models to Examine the Mutual Influence of Financial Markets in the World), *Equilibrium*, Wydawnictwo Naukowe UMK, Toruń, 1(2), 61–68.

Engle, R. (1982), Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of UK Inflation, *Econometrica,* 50, 987–1008.

Fiszeder, P., (2009), *Modele klasy GARCH w empirycznych badaniach finansowych* (*GARCH Models in Empirical Studies of Financial*), Wydawnictwo UMK, Toruń.

Osińska, M. (2006), *Ekonometria finansowa* (*Financial Econometrics*), PWE, Warszawa.

Piontek, K. (2006), Wyzwania praktyczne w modelowaniu wielowymiarowych procesów GARCH (The Practical Challenges in the Modeling of Multivariate GARCH Processes), *Taksonomia (Taxonomy)*, Akademia Ekonomiczna, Wrocław, No. 13.

Yang, W. (2001), *M-GARCH Hedge Ratios and Hedging Effectiveness in Australian Futures Markets*, Edith Cowan University.

Badanie współzależności rynków kapitałowych i walutowych z zastosowaniem modeli MGARCH

Z a r y s t r e ś c i. W artykule podjęto próbę zbadania wzajemnych relacji pomiędzy wybranymi indeksami giełdowymi, relacji pomiędzy kursami walut, a następnie określono kierunek przepływu informacji pomiędzy rynkami kapitałowym i walutowym. Do oszacowania współzależności instrumentów inwestycyjnych posłużył wielowymiarowy model klasy GARCH. Praca jest kontynuacją poprzednich badań autora związanych z taksonometryczną klasyfikacją giełd na świecie.

S ł o w a k l u c z o w e: wielowymiarowy model GARCH, analiza współzależności, giełdy papierów wartościowych, waluty.