An attempt at a typology of karst lakes in the Połaniec Basin (Małopolska Upland)

Michał Stachura* [ID] [1a], **Dariusz Wieczorek** [ID] [2b], **Artur Zieliński** [ID] [1c]

[1] Jan Kochanowski University in Kielce, Poland, [2] Holy Cross Society of the Friends of Geological Sciences in Kielce and Polish Geological Institute - National Research Institute, Holy Cross Branch in Kielce, Poland

* Correspondence: Institute of Mathematics, Jan Kochanowski University in Kielce, Poland. E-mail: mista@ujk.edu.pl

[ID] [a] https://orcid.org/0000-0002-0115-3522, [b] https://orcid.org/0000-0001-6612-5349, [c] https://orcid.org/ 0000-0003-1672-7776

Abstract. The main goal of the paper is to attempt a typology of karst lakes in the Połaniec Basin (Małopolska Upland). The typology was conducted on the basis of a dependence analysis of several essential morphometric parameters of lake basins. The considered data comprised 23 lakes with respect to 15 morphometric features. The correlation analysis, mainly of a group of lakes located in single karst sinkholes, revealed that the length and width of basins are strongly correlated. It is also noticeable that basin shape determines lake volume, even though pools of similar water volume may differ in area. Moreover, an increase in the maximum depth of basins does not necessarily imply any increase in volume. Likewise, there is no prevalent dependence between basin area and maximum depth. The cluster analysis, among reasonable indications, generally identified a division of the considered lakes into two sets. One of the sets comprises Duży Staw and Dziki Staw, while the other consists of all the other lakes. Less frequent divisions into three indicated Duży Staw, Dziki Staw, and Czwarty Staw as the leading lakes. Divisions into 19–22 clusters were also suggested, but this does not seem to be reliable. As a consequence, the cluster analysis showed that Duży Staw and Dziki Staw stand out the most from the other lakes. This remainder constitutes rather close to each other, but not an ideally uniform group of lakes.

Introduction

Lakes of above 1 ha are not common in the Polish Uplands (Choiński 1995). They are found mainly in Western Polesie, also known as Polesie Lubelskie (Wilgat et al. 1991; Michalczyk 1998; Chmielewski 2001). These lakes are considered karstic (carbonate karst). Another interesting place is the Połaniec Basin on the Małopolska Upland, where karst landforms were formed on gypsum substrates –

gypsum karst (Flis 1954; Chwalik-Borowiec 2013; Zieliński 2013). Lakes that develop as a result of gypsum karst are interesting, though they are still little researched both in Poland and elsewhere in the world.

In this study, karst lakes from Połaniec Basin have been analysed statistically, and then methods of statistical classification have been applied in order to attempt a typology of these lakes, to make a comparative analysis to establish whether they are related to each other and, if so, to what extent.

In other words, whether we can treat them as a uniform group. The study involves selected lakes in the following three areas: (1) the Staszów region, (2) the area east of Chmielnik, near Wola Zofiowska, and (3) between Chmielnik and Pińczów, near Szarbków (Fig. 1). Permanent lakes of the biggest surface area in their group of lakes, and additionally those which have bathymetric planes, were selected for these analyses.

Areas and lakes studied

The study area is situated in the mesoregion of the Połaniec Basin in the east of the Nida Basin macroregion – both lie on the Małopolska Upland subprovince (Kondracki 2013).

So-called covered, reproduced karst is a characteristic feature of the Połaniec Basin northern part. This is due to karstifying layers of gypsum, which are usually covered by younger formations, these being Miocene clays of the Grabowiec and Krakowiec beds, deposits of marginal facies (detrital), and Pleistocene sands and tills (Senkowicz 1958; Walczowski 1968; Romanek 1982). In gypsum

environments karst lakes are quite common, but dry karst landforms, especially sinkholes, are even more numerous.

The biggest concentration of karst lakes is found in the Staszów region. They are also present in the vicinity of Jarząbki and Szarbków. Some of these lakes are so close to each other that they form visible groups.

The statistical analysis involved characteristic data for 23 lakes (Table 1, Fig. 2). Most of them are situated in the Staszów region, while one lake is near Szarbków, and one is near Wola Zofiowska. The original statistical material was prepared by Zieliński (2013) and pertained to 25 lakes. In this study, that part of the data (variables) established and measured for 23 lakes has been used that provided a uniform database. Morphometric parameters of lakes basins were calculated using the formulas included in the works of Skowron (2004) and Choiński (2007).

With regard to their form and basin shape, these lakes can be divided into (Zieliński 2013):

- lakes in single karst sinkholes, for example: Donica Lake, Łajba Lake, Drugi Staw, "Bez Nazwy" Lake, Pleban Lake;

Fig. 1. Location of study areas on the contour map of Poland (A). Location of more numerous groups of lakes (B). Red dashed lines – boundaries of macroregions (Kielce Upland, Nida Basin, Sandomierz Basin). Black dash-dotted lines – boundaries of mesoregions (JP - Jędrzejów Plateau; SF - Szydłów Foothills, HCM – Holy Cross Mountains, SU – Sandomierz Upland, Połaniec Basin, Pińczów Hump, Wodzisław Hump, Nida Valley, Solec Basin, VL- Vistula Lowland).

- lakes in uvalas, for example: Duży Staw, Przedpole Lake, Rozlany Staw, Kacze Lake, Jasny Staw;

- lakes in the bottoms of karst valleys that cut the surface of the Paleogene peneplanation surface, for example: Dziki Staw.

These lakes can also be grouped in the following way:

- those whose waters have no contact with karst formations, e.g. lakes in the Golejowskie Forests near Staszów, lakes near Jarząbki and lakes situated round Pleban Lake;

- those whose waters have contact with karstifying rock, e.g. some unnamed lakes in the Szaniec Plateau (Zieliński 2013).

The surface area of the lakes studied here is relatively small, ranging from 0.08 to 3.63 hectares (Zieliński 2013). Lakes situated in isolated karst sinkholes are especially small. Their small size notwithstanding, they are among the deepest natural lakes in the Świętokrzyski region. It is worth noting that lakes in karst formations do not always fill these formations to their full capacity. The present focus is on analysing lakes, without referring to the karst formations themselves.

Methods

Parameters used in the analysis

For the statistical analyses, the following data have been collected from a previous study by Zieliński (2013): data concerning the lakes' morphometry (surface area, length, maximum width, mean width, length ratio, length of shoreline, development of shoreline length) as well as parameters of lake basin (volume, maximum depth, mean depth, relative depth, ratio of basin shape, i.e. depth index, mean bottom slope, i.e. mean bottom inclination, index of lake basin permanence, i.e. basin permanence index). Altogether, information about 23 lakes was used according to 15 variables (Table 1). The table shows the lakes in alphabetical order. For ease of search, the ordinal numbers in the first column of Table 1 refer to the lake numbers in Figure 2.

Correlation analysis

As a preliminary investigation of the relationship between variables, simple correlation analyses of pairs of variables was carried out. Since the type of correlation is unknown, three different coefficients are calculated in each case – Pearson's, Spearman's and Kendall's. The strength of correlation is assessed following the interpretation proposed by Evans (1996).

Cluster analysis

Inasmuch as some variables are direct transformations of other variables (e.g. the length ratio of a lake is calculated by dividing its length by its maximum width), some of the data reflect close parameters (e.g. 'maximum width' and 'mean width' pair of variables), so the equivalent treatment of all the 15 variables is debatable.

For this reason, the cluster analysis (Gatnar and Walesiak 2004; Walesiak and Gatnar 2009) was carried out three separate times on the basis of a different base pair each time: variables used in cluster analysis as well as the way the distance between objects is determined. The base pairs are as follows:

(P1) all variables taken into account; distance determined on the basis of standardised values of all variables;

(P2) part of variables taken into account; distance determined on the basis of values of these variables;

(P3) all variables taken into account; distance determined on the basis of values that were standardised and then weighted (for the variables).

In each case, the analysis was based on a correspondingly multi-dimensional Euclidean metric, as distances between objects.

In the case of (P1) base pair, agglomerative clustering was performed, followed by divisive clustering using the *hclust*, *agnes*, *diana* functions of R Package (R Core Team 2015) – with "average distance" taken as the distance measure between successively created (connected/divided) clusters. In parallel, Caliński-Harabasz, silhouette, Dunn, and GAP indices, as well as the Pearson gamma for partitions into clusters, with were determined
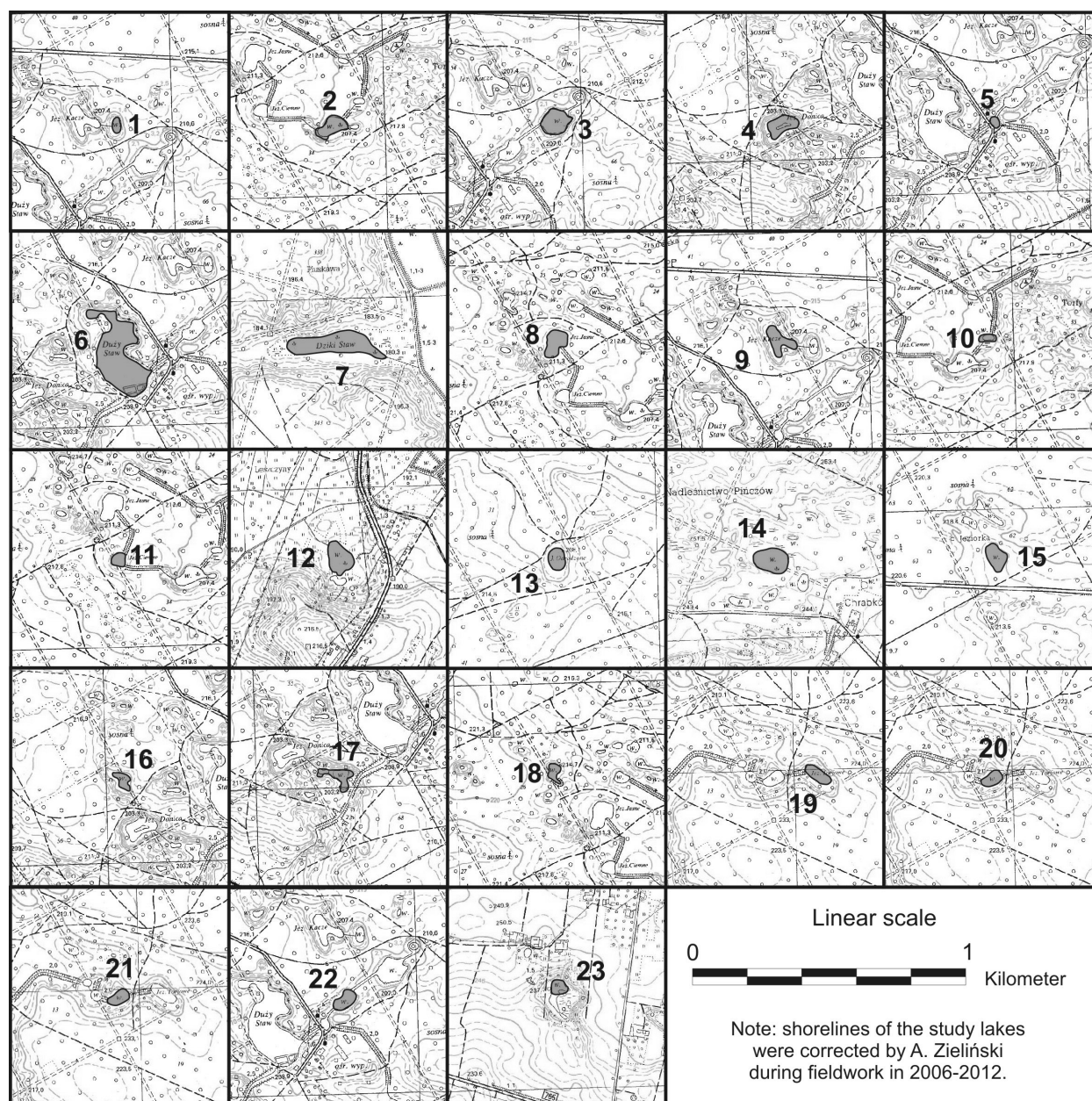
Fig. 2. Location of the lakes studied in sections of topographical maps (lakes numbering according to Table 1)

in order to identify their optimum number (Korzeniewski 2014) with use of the *cluster.stats* function (Hennig 2015).

In the case of the (P2) base pair, all the same procedures were carried out. Variable typing was performed by heuristic identification of noisy variables, using the *HINoV.Mod* function of the clusterSim package with the following settings: partition into two clusters, Euclidean metric, distance between groups as mean distance (Walesiak and Dudek 2016). This method identified the following variables as confounders: "mean bottom slope",

"mean depth", "development of shoreline", "length ratio", "ratio of basin shape", and, as a consequence, these variables were omitted. (The confounding variables are listed above in order of least to most insignificant.) The remaining variables, as was the case in (P1), served as the basis for agglomerative and divisive clustering. Likewise, indices were determined as with (P1).

In turn, in the case of the (P3) base pair, before carrying out the same procedure, an attempt at setting weights of all the variables was undertaken. To do this, roles of variables and objects were

reversed and cluster analysis for variables was performed: agglomerative clustering (*hclust* function), with the distance between variables calculated as, where is the value of Kendall's tau coefficient for a given pair of variables, and with average distance between created groups. After that, values of "height", i.e. the distance of successively included single variables to the groups already created, were raised to the 0.25 power in order to weaken their excessive differentiation, and accepted as weights for variables.

Preliminary investigation of relationship between pairs of variables

Simple correlation analyses of pairs of variables showed quite a noticeable relationship between the

following (and very strong relationships for Pearson's coefficient in the sense of Evans interpretation [Evans 1996]):

- lakes' length and their maximum width (0.829, 0.751, 0.567: values of Pearson's linear correlation coefficient, Spearman's rank correlation coefficient, and Kendall's tau coefficient respectively);

- lakes' area and their volume (0.892, 0.803, 0.652);

- lakes' area and their length (0.987, 0.954, 0.851);

- lakes' area and their shoreline length (0.980, 0.915, 0.758);

- lakes' length and their shoreline length (0.986, 0.904, 0.755);

- relative depth ratio of lakes and their mean bottom slope (0.841, 0.820, 0.693).

Such relationships are most noticeable in the group of lakes situated in single karst sinkholes.

Table 1. Morphometric parameters of lakes and lake basins

No	Name	Area (P) [m²]	Volume (V) [m³]	Length (D) [m]	Maximum width (W) [m]	Mean width $(W_m=P/D)$ [m]	Relative depth ratio $(C_R=H_{max}/W_m)$	Length ratio $(\lambda=D/W_m)$
1.	"Bez Nazwy"	2,100	2,600	54	49	38.9	0.0913	1.39
2.	Ciemne	4,900	18,500	116	72	42.2	0.0492	2.75
3.	Czwarty Staw	9,500	44,100	143	103	66.4	0.1552	2.15
4.	Donica	5,900	14,100	120	75	49.2	0.1598	2.44
5.	Drugi Staw	1,840	4,000	68	44	27.1	0.1219	2.51
6.	Duży Staw	36,300	134,000	425	156	85.4	0.1241	4.98
7.	Dziki Staw	26,000	32,000	362	105	71.8	0.1039	5.04
8.	Jasny Staw	7,600	18,800	127	84	59.8	0.0549	2.12
9.	Kacze	6,100	2,600	120	76	50.8	0.1145	2.36
10.	lake on E from Ciemne	800	1,200	31	27	25.8	0.0669	1.20
11.	lake on W from Ciemne	1,800	1,800	61	42	29.5	0.0334	2.07
12.	Łajba	8,200	14,000	123	95	66.7	0.0568	1.85
13.	Odrodzone	5,700	17,000	96	78	59.4	0.1268	1.62
14.	Pleban	8,600	13,900	147	81	58.5	0.1255	2.51
15.	Przeciwpożarowe	6,600	8,400	125	72	52.8	0.1215	2.37
16.	Przedpole	3,000	4,600	82	77	36.6	0.0769	2.24
17.	Rozlany Staw	2,700	2,300	78	79	34.6	0.1090	2.25
18.	Szyja	2,500	3,600	109	44	22.9	0.0645	4.75
19.	Torfowe I	5,100	14,000	122	53	41.8	0.1610	2.92
20.	Torfowe II	3,900	9,500	96	67	40.6	0.0712	2.36
21.	Torfowe III	2,100	4,200	58	46	36.2	0.1511	1.60
22.	TrzeciStaw	4,400	14,400	95	57	46.3	0.0581	2.05
23.	Zofiówka	3,600	4,800	83	75	43.4	0.0576	1.91

Table 1. Morphometric parameters of lakes and lake basins - continue

No	shoreline length (L) [m]	development of shoreline length $(K_l=L/(2\sqrt{\pi P}))$ [m·ha⁻¹]	maximum-depth (H_{max}) [m]	mean depth $(H_m=V/P)$ [m]	relative depth $(H_r=H_{max}/\sqrt{P})$	ratio of basin shape $(W_g=H_m/H_{max})$	mean bottom slope $(\alpha=atan(h\Sigma l/P))$ [°]	index of basin permanence $(BPI=V/L)$ [m³·m⁻¹]
1.	169	1.04	2.6	1.24	0.0567	0.4762	9	15.38
2.	314	1.27	6.8	3.78	0.0971	0.5552	20	58.92
3.	368	1.07	8.1	4.64	0.0831	0.5731	14	119.84
4.	361	1.33	6.1	2.39	0.0794	0.3918	15	39.06
5.	189	1.24	4.2	2.17	0.0979	0.5176	18	21.16
6.	1095	1.62	7.8	3.69	0.0409	0.4733	9	122.37
7.	860	1.50	2.4	1.23	0.0149	0.5128	4	37.21
8.	387	1.25	4.6	2.47	0.0528	0.5378	12	48.58
9.	413	1.49	2.5	0.43	0.0320	0.1705	7	6.30
10.	102	1.02	3.9	1.50	0.1379	0.3846	18	11.76
11.	167	1.11	2.1	1.00	0.0495	0.4762	10	10.78
12.	350	1.09	4.3	1.71	0.0475	0.3971	8	40.00
13.	278	1.04	6.8	2.98	0.0901	0.4386	12	61.15
14.	385	1.17	3.4	1.62	0.0367	0.4754	7	36.10
15.	343	1.19	3.0	1.27	0.0369	0.4242	4	24.49
16.	278	1.43	3.8	1.53	0.0694	0.4035	11	16.55
17.	327	1.78	1.9	0.85	0.0366	0.4483	11	7.03
18.	269	1.52	2.5	1.44	0.0500	0.5760	14	13.38
19.	289	1.14	5.3	2.75	0.0742	0.5179	12	48.44
20.	256	1.16	5.1	2.44	0.0817	0.4776	14	37.11
21.	190	1.17	4.4	2.00	0.0960	0.4545	16	22.11
22.	236	1.00	7.4	3.27	0.1116	0.4423	17	61.02
23.	238	1.12	2.5	1.33	0.0417	0.5333	16	20.17

Source: own materials, based on Zieliński (2013) $\sum l$ – sum of length of all the isobaths; h – contour line cut

The shape of the karst basin correlates to the volume of water in a lake. However, lakes of a similar volume may differ considerably in terms of their surface area, which is the case in, for example, Trzeci Staw, Łajba and Pleban lakes.

Interestingly, the increase in the maximum depth of the analysed lakes does not always result in a noticeable increase in their volume (0.558 – Evans' moderate, 0.692, 0.564). This is also the case of the dependence between the lake surface area and its maximum depth (0.295 – weak, 0.349, 0.232). However, the dependence between lake maximum depth and its mean depth is good (0.951 – very strong, 0.962, 0.849): an increase in maximum depth usually leads to an increase in mean depth. The relationship between the lakes' mean depth and their mean bottom slope is poor (0.4444 – moderate, 0.486, 0.312).

In turn, the size of lake surface has a negative relation with its mean bottom slope (-0.448 – Evans' moderate, -0.509, -0.349). Lakes of area of up to 0.5 ha have the biggest mean bottom slope.

Results of cluster analysis

In the case of the (P1) base pair, the number of clusters was shown as 2 (20% indications), 3 (20% indications), 20 (20% indications), and 22 (40% indications). No other numbers of clusters were shown. This suggests that the division into two or three clusters is the most justifiable among reasonable divisions. Interestingly, all the clustering methods used determined the same two clusters: one consisting of Duży Staw and Dziki Staw, the other comprising all the remaining lakes. These divisions
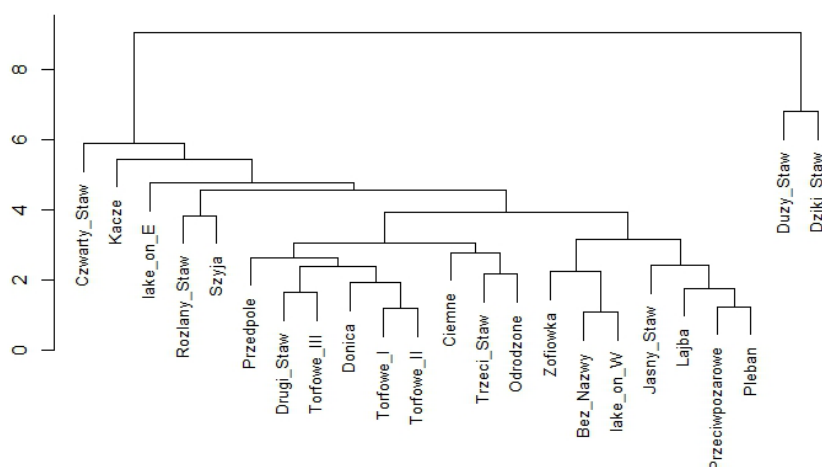
Fig. 3. Dendrogram of lakes for (P1) base pair using 'Hclust' function
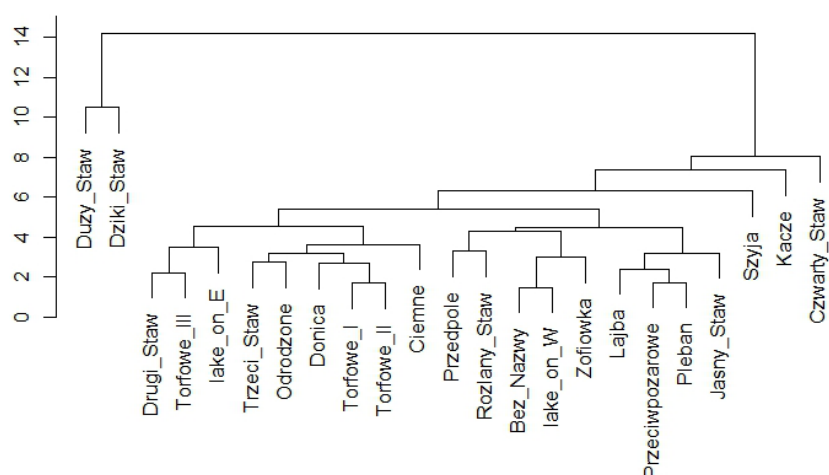

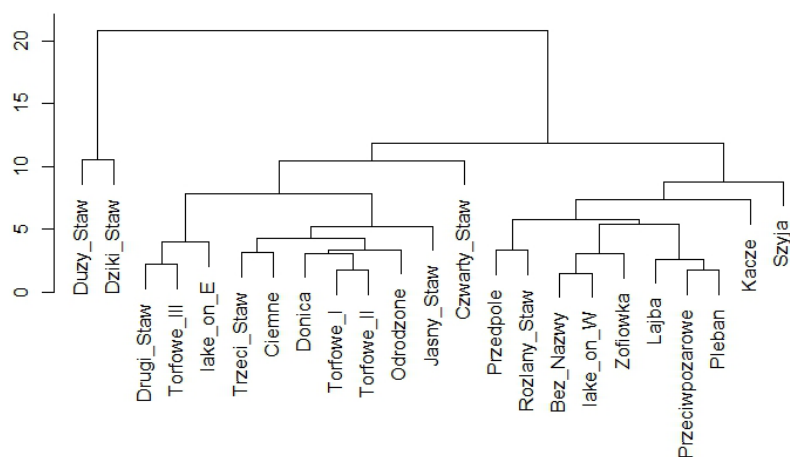Fig. 4. Dendrogram of lakes for (P1) base pair using 'Agnes' function

Fig. 5. Dendrogram of lakes for (P1) base pair using 'Diana' function

are presented in the following dendrograms (Figs 3–5):

In the division into three clusters, agglomerative methods resulted in identifying Duży Staw and Dziki Staw as two separate clusters, and all the remaining lakes as the third cluster. In turn, divisive methods put Duży Staw and Dziki Staw into one group, and the remaining lakes into two groups of a roughly similar number of lakes, with Czwarty Staw and Szyja Lake, respectively, standing out the most.

It is worth noting that the option of dividing the lakes into 20 or 22 clusters may suggest that the set of the lakes analysed here (albeit with the exception of Duży Staw and Dziki Staw and perhaps also Czwarty Staw and Szyja Lake) is still not homogeneous enough for the lakes to be considered strongly similar to each other, despite being quite homogeneous in terms of the variables that describe the lakes.

In the case of the (P2) base pair, the number of clusters was shown as 2 (20% indications), 3 (20% indications), 19 (40% indications), and 22 (20% indications). No other numbers of clusters were shown.

In the case of the two-cluster division, all the clustering methods used determined the same two clusters: one consisting of Duży Staw, and the other comprising all the remaining lakes with Dziki Staw, and Czwarty Staw, consecutively, 'standing out' the most.

The three-cluster division corresponds to that of (P1): consistent division into: 1. Duży Staw, 2. Dziki Staw, 3. all other lakes. Interestingly, in the last group, Czwarty Staw, mentioned in (P1), is the one that also 'stands out' most.

It is necessary to point out that the omission of a number of variables – as was the case for the (P2) base pair – was solely due to the specificity of the heuristic identification of noisy variables method, and, in the authors' opinion, this led to the omission of quite important variables that describe lakes, especially the ratio of basin shape.

Finally, the case of the (P3) base pair starts with a reversed clustering problem. Thus, the results of the variable grouping is shown in the dendrogram in Figure 6. It is worth noting that the biggest weight is given to the 'ratio of basin shape' variable, which was been eliminated by the heuristic method as the most insignificant. The current weight given to this variable seems to fully reflect the importance intuitively ascribed to it.

In this case as well, the numbers of clusters were similar, and were as follows: 2 (30% indications), 4 (10% indications), 20 (40% indications), and 22 (20% indications). The division into two clusters, the most justifiable one among the reasonable divisions, determined the same two clusters as with (P1) and (P2). In turn, the division into four clusters via divisive methods identify: (1) Duży Staw and Dziki Staw, (2) Kacze Lake and Rozlany Staw, (3) Czwarty Staw with a group of 9 other lakes, and (4) Szyja Lake with a group of the 8 remaining lakes.

Further, similar procedures were performed again for (P1), (P2) and (P3), this time with the Euclidean distance replaced by the Manhattan distance. In these cases, very similar results were
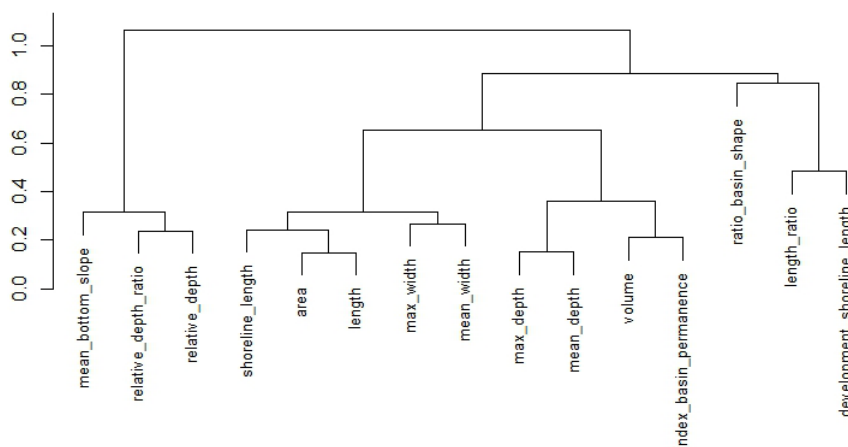

Fig. 6. Dendrogram of variables for (P3) base pair

obtained, both for the suggested numbers of clusters and for the object grouping itself. The dominant reasonable divisions into two clusters are in each case identical to those with the Euclidean distance. The divisions into three clusters are also of the same types as before, although the division into (1) Duży Staw, (2) Dziki Staw, (3) the rest of the lakes (with Czwarty Staw 'standing out' most) is absolutely predominant. There were no indications for divisions into four clusters.

Similarly, divisions into 19, 21 and 22 clusters were suggested. Thus, this time, too, we can see some kind of polarisation of the numbers of suggested clusters.

The cluster analysis described above, and intentionally performed in many various ways, allows us to conclude that two lakes: Dziki Staw and Duży Staw clearly stand out from the other lakes. The remaining lakes are a quite (though not ideally) homogeneous group. This homogeneity is disturbed by the suggested divisions into a big number of clusters, as well as by some lakes standing out of this group occasionally and forming separate clusters.

Such a perception of the objects studied is also confirmed by the way the points representing lakes are distributed in the system of coordinates for three primary principal components (*Principal Component Analysis*, PCA) determined on basis of the set of variables analysed. Obviously, the point distribution is dependent on choosing (P1), (P2) and (P3) as a starting point. Figure 7 shows (P1) the point distribution with the Euclidean distance, with PC1 and PC2 axes representing the first and second principal components, respectively. The third component is shown by the size of the points: the smaller the point, the deeper it is from the PC1–PC2 plane. The points are numbered according to the lake numbers in Table 1.

Discussion

The analysed lakes from the Połaniec Basin area are generally small objects, and their characteristic feature is a relatively large depth and circular shape (Fig. 8). These parameters differ from other water bodies in both Świętokrzyskie and other regions.

Geological determinants undoubtedly influence the unique specifics of the objects.

Even smaller natural lakes are known from the area of the Kolbuszowa Plateau, from the Grodzisko Górne and Grodzisko Dolne near Leżajsk (Wojtanowicz and Jóźwiakowska 1997). They occur primarily on the highest levels of the plateau, but also on its slopes, within denudational valleys and also in the valley of the Leszczynka stream. There are 82 lakes near Grodzisko Górne and Grodzisko Dolne, but only in two cases does their longer axis exceed 100 m. Most are 10–80 m long and 10–40 m wide (Fig. 9). They are most often oval and, less frequently, circular. The lakes are relatively shallow: 20 of them are up to 1.0 m deep, 24 are 1.0–1.5 m deep, and 26 are 1.5–2.0 m deep. One lake is about 3.5 m deep because it was dredged by the owner of the ground on which it is located. Those lakes, which developed on the cover of silty deposits, i.e. they are located on the top and slopes of the Kolbuszowa Plateau, were formed in the late LGM, together with degradation of long-term permafrost in the ground resulting from thermal karst (Wojtanowicz 1997). Research on these basins has shown that only 20 of them have water throughout the year (Jóźwiakowska 1997). Some of the lake basins (18) accumulate water only at the beginning of spring. In the rest of the lakes there have been anthropogenic changes, as a result of which 39 are already dried up and 5 have been buried.

What in our climate conditions can be considered as unique is common in other geographic locations, such as northern Norway (cf. Svensson 1969). In the circumpolar climates zone, in the subpolar climate type, the occurrence of long-term permafrost provides the formation of characteristic oval lakes. The dimensions of these aquatic areas are often small, of the order of several dozen metres. They are shallow, at usually up to 1 m deep. The occurrence of earth embankments around their bowls is characteristic.

In recent years, dune lakes have been described in the Świętokrzyskie region, and their assemblages called an (aeolian) lakeland (Jaśkowski and Sołtysik 2003). This name was used not so much in a regional sense, but in the sense of a landscape. The surface of these basins are sometimes larger than the described karst lakes, but their depth is small, at up to only 2 m.
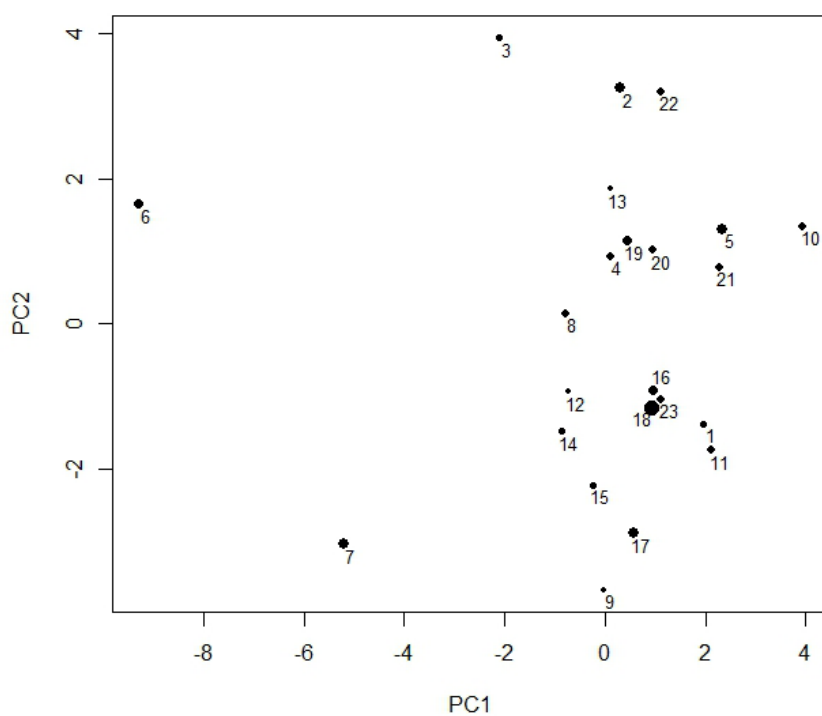
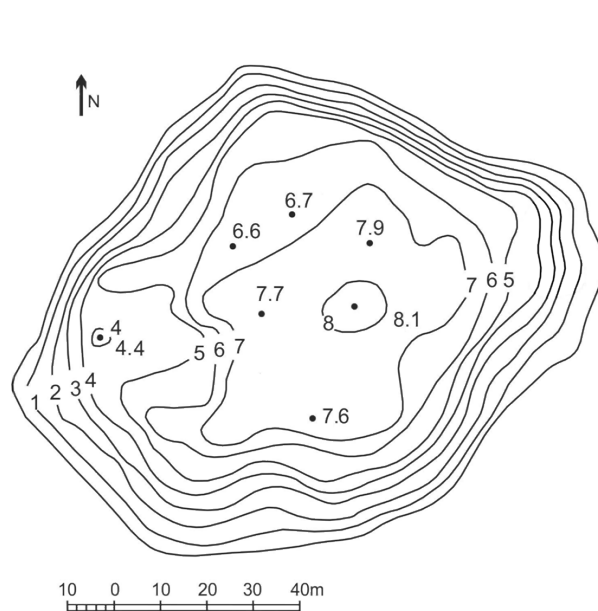Fig. 7. Lake distribution in the system of three principal components

Fig. 8. Bathymetric plan of Czwarty Staw, based on Zieliński (2013); numbers show depth in meters
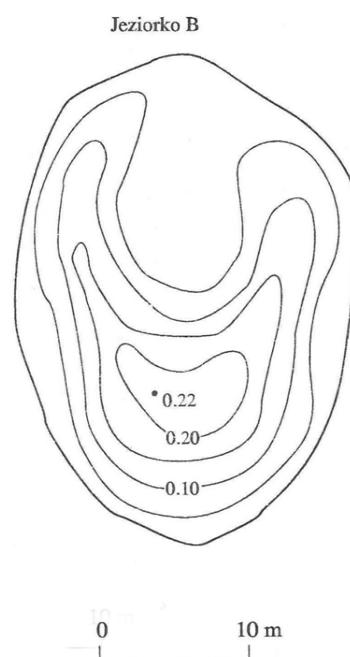
Fig. 9. Bathymetric plan of Jeziorko B, based on Jóźwiakowska (1997); numbers show depth in meter

Much larger aquatic areas occur in northern Poland, in lakelands formed within the glacial uplands (Choiński and Ptak 2009). Lakes there (e.g. within the Greater Poland-Kujawy Lakeland, the Mazurian Lakeland and the Pomeranian Lakeland) cover hundreds of hectares, and have maximum depths of tens of metres, average depths of a few to several metres, and volumes from several hundred thousand to several tens of millions of cubic metres. The shapes of the bowls of these lakes are varied. There are compact bowls (melt lakes), elongated bowls (gutter lakes), and those of mixed shape (diversified origin). Recent research indicates water loss in lakes and, consequently, changes in the coastline, e.g. the formation of peninsulas and islands (Choiński et al. 2016).

Of interest are the coastal lakes Jamno and Bukowo (Choiński et al. 2014). They are of ca. 2,200–1,600 ha, with water volumes of ca. 38–28 million m3, but at the same time are very shallow, averaging 1.7 m. Over the last century, their area decreased by 8–14%, and their water volume by 6–18%, with little loss of depth.

It would seem interesting to compare the lakes in other regions of Poland of different origin with the karst lakes of the Połaniec Basin in terms such as presented in this work. For this, more data should be collected, similar to those provided by Zieliński (2013). For the same reason, the authors cannot compare the discussed reservoirs with other sites located in areas of similar geological structure.

This article also provides data on the morphometry of karst mire lakes studied by Zieliński (2013).

Conclusions

The lakes selected for the statistical analysis are situated within the same mesoregion – the Połaniec Basin. They are in an area of fairly uniform geological structure and climate conditions, hence their "origin and form"?

Correlation analysis showed a clear relationship between lake length and its width. Such dependences are the most visible in the group of lakes in single karst sinkholes. The analysis also shows that lake basin shape is a decisive factor in the volume of water in a given lake. However, lakes of a similar volume may differ in terms of area. What is more, an increase in maximum depth does not always result in an increase in volume. This is also the case of the dependence between lake area and its maximum depth.

The most frequent reasonable indication of the cluster analysis was for grouping lakes within two sets. One of them consists of Duży Staw and Dziki Staw, and the other set comprises the remaining 21 lakes. In this method, in the division into three clusters (the division occurring less frequently), Duży Staw, Dziki Staw, and Czwarty Staw stood out the most, creating either two groups or three separate groups. In turn, the divisions of the lakes analysed into 19–22 clusters occurred more often, but it does not seem to be reasonable to take into account such divisions, since it was nearly the same as the number of lakes themselves.

The cluster analysis performed in many various ways shows that Dziki Staw and Duży Staw stand out the most from the other lakes. The other lakes are close to each other, but they do not form an ideally uniform group.

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