CHARACTERIZATION OF REGULARITY SETS
FOR FAMILIES OF SEQUENCES OF MATRICES

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ABSTRACT. We show that the regularity set of any family of sequences of matrices is an $F_{\sigma\delta}$-set. Moreover, for any $F_{\sigma\delta}$-set containing zero, we construct a family of sequences of matrices having this regularity set.

1. Introduction

We consider 1-parameter families $A(\theta) = (A_n(\theta))_{n\in\mathbb{N}}$ of sequences of invertible $q \times q$ matrices $A_n(\theta)$ varying continuously with $\theta \in \mathbb{R}$ for each $n \in \mathbb{N}$. For simplicity, we shall refer to them simply as 1-parameter families (of sequences of matrices). Each sequence $A(\theta)$ defines a nonautonomous dynamics with discrete time. Our aim is to characterize the regularity sets of these families, that is, the sets of parameters for which the dynamics of the corresponding sequence is (Lyapunov) regular. The main novelty of our work is the consideration of arbitrary sequences of matrices, possibly unbounded together with their inverses.

1.1. The notion of regularity. Going back to Lyapunov's thesis [14], the notion of regularity plays an important role in the stability theory of differential equations and dynamical systems. This is mainly due to the fact that the nonuniform exponential stability of a linear dynamics need not persist under small nonlinear perturbations. More precisely, the notion of regularity allows one