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SMALL PERTURBATIONS OF ROBIN PROBLEMS DRIVEN BY THE *p*-LAPLACIAN PLUS A POSITIVE POTENTIAL

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ABSTRACT. We consider a quasilinear Robin problem driven by the *p*-Laplacian plus a positive potential and with a small perturbation. We assume that the main term in the equation has an Ekeland structure but we do not suppose any growth condition for the perturbation term. Applying variational methods, we prove the existence of at least one nontrivial weak solution.

1. Introduction

In this paper, we study the following quasilinear Robin p-Laplace problem with small perturbation given by

(1.1)
$$\begin{cases} -\Delta_p u + V(x)u = a(x)|u|^{q-1}u + \lambda g(x,u) + f(x) & \text{in } \Omega, \\ |\nabla u|^{p-2}\nabla u \cdot \nu + \beta(x)|u|^{p-2}u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subseteq \mathbb{R}^N$ (N > 2) is a bounded domain with a C^2 -boundary $\partial \Omega$, $p \ge 2$, λ is a real parameter, 0 < q < p - 1 and $\nu(x)$ denotes the outer unit normal of

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