

SECOND-ORDER BOUNDARY ESTIMATES FOR LARGE POSITIVE SOLUTIONS TO AN ELLIPTIC SYSTEM OF COMPETITIVE TYPE

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ABSTRACT. In this paper, we study the second-order boundary asymptotic behaviour for large positive solutions to an elliptic system of competitive type. First, we derive a second-order estimate to a related single weighted equation with boundary blow-up data. Then, by relaxing the system and iterating the estimate of the single equation, we establish second-order estimates of the solutions.

1. Introduction and main results

Consider large solutions of the following semilinear elliptic system

$$(1.1) \quad \begin{cases} \Delta u = u^p v^q & \text{in } \Omega, \\ \Delta v = u^r v^s & \text{in } \Omega, \\ u = +\infty, v = +\infty & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain of class $C^{2,\mu}$ for some $0 < \mu < 1$. The parameters $p, s > 1$ and $q, r > 0$. The condition $u = +\infty, v = +\infty$ on $\partial\Omega$ is defined in the sense of that $u(x) \rightarrow +\infty$ and $v(x) \rightarrow +\infty$ as $\text{dist}(x, \partial\Omega) \rightarrow 0$.

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