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# UNIQUENESS OF SOLUTIONS FOR NONLINEAR DIRICHLET PROBLEMS WITH SUPERCRITICAL GROWTH

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Abstract. We are concerned with Dirichlet problems of the form

$$\operatorname{div}(|Du|^{p-2}Du) + f(u) = 0 \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega,$$

where $\Omega$ is a bounded domain of $\mathbb{R}^n$, $n \geq 2$, $1 < p < n$ and $f$ is a continuous function with supercritical growth from the viewpoint of the Sobolev embedding. In particular, if $n = 2$ and $\gamma\colon [a,b] \to \mathbb{R}^2$ is a smooth curve such that $\gamma(t_1) \neq \gamma(t_2)$ for $t_1 \neq t_2$, we prove that, for $\varepsilon > 0$ small enough, there exists a unique solution of the Dirichlet problem in the domain $\Omega = \Omega_\varepsilon^\Gamma = \{(x_1,x_2) \in \mathbb{R}^2 : \operatorname{dist}\big((x_1,x_2),\Gamma\big) < \varepsilon\}$, where $\Gamma = \{\gamma(t) : t \in [a,b]\}$. Moreover, we extend this uniqueness result to the case where $n > 2$ and $\Omega$ is, for example, a domain of the type

$$\Omega = \widetilde{\Omega}_{\varepsilon,s}^\Gamma = \{(x_1,x_2,y) : (x_1,x_2) \in \Omega_\varepsilon^\Gamma,\ y \in \mathbb{R}^{n-2},\ |y| < s\}.$$

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