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ON NONLINEAR SCHRÖDINGER EQUATIONS WITH ATTRACTIVE INVERSE-POWER POTENTIALS

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Abstract. We study the Cauchy problem for nonlinear Schrödinger equations with attractive inverse-power potentials. By using variational arguments, we first determine a sharp threshold of global well-posedness and blow-up for the equation in the mass-supercritical case. We next study the existence and orbital stability of standing waves for the problem in the mass-subcritical and mass-critical cases. In the mass-critical case, we give a detailed description of the blow-up behavior of standing waves when the mass tends to a critical value.

1. Introduction and main results

We consider the Cauchy problem for nonlinear Schrödinger equations with attractive inverse-power potentials

$$\begin{cases} i\partial_t u + \Delta u + |x|^{-\sigma}u = \pm|u|^{\alpha}u, \quad (t,x) \in \mathbb{R}\times\mathbb{R}^d, \\ u(0) = u_0, \end{cases} \tag{1.1}$$

where $u\colon \mathbb{R}\times\mathbb{R}^d \to \mathbb{C}$, $u_0\colon \mathbb{R}^d \to \mathbb{C}$, $0 < \sigma < \min\{2,d\}$ and $\alpha > 0$. The plus (resp. minus) sign in front of the nonlinearity corresponds to the defocusing (resp. focusing) case.

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Key words and phrases. Nonlinear Schrödinger equation; inverse-power potential; standing waves; stability; global well-posedness; blow-up.

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