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SPACE-TIME DECAY ESTIMATES OF SOLUTIONS TO 3D INCOMPRESSIBLE VISCOUS CAMASSA–HOLM EQUATIONS

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Abstract. In this paper, based on the parabolic interpolation inequality and inductive argument, we study the space-time decay estimates of higher-order time and spatial derivatives of strong solutions for the 3D incompressible viscous Camassa–Holm equations provided that the initial datum is well localized.

1. Introduction

Consider the viscous Camassa–Holm (Navier–Stokes-α) equations in $\mathbb{R}^3$:

$$\begin{cases} \partial_t v + u \cdot \nabla v + \sum_j v_j \nabla u_j + \nabla \pi - \nu \Delta v = 0, \\ u - \alpha^2 \Delta u = v, \\ \nabla \cdot u = \nabla \cdot v = 0, \\ v(x,0) = v_0(x), \end{cases} \tag{1.1}$$

where $v = (v_1, v_2, v_3)$, the fluid velocity field, $u = (u_1, u_2, u_3)$, the 'filtered' fluid velocity, and π, the pressure, are the unknowns. $\nu > 0$ is the constant viscosity,

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