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DISSIPATIVE SOLVABILITY OF JEFFREYS–OLDROYD- α MODEL

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ABSTRACT. We study the initial-boundary value problem for the so-called alpha model of Jeffreys–Oldroyd fluids motion in 2D and 3D dimensions. In this paper the global in time existence of dissipative solutions to this problem is obtained. For this the topological approximation method to the initial-boundary value problem is applied.

1. Introduction

At the recent thirty years alpha models of hydrodynamics are intensively investigated and popularized. These models in comparison with the original models represent a kind of approximation, which depends on the parameter α . Alpha model differs from the original one by replacement in a number of terms the velocity function v by a smoother function u connected with v. The idea of using such approximations for the study of original models first arose in the work by J. Leray [14]. Later, based on the idea the theory of alpha models, in which the vector-valued velocity function v is replaced in a series of terms by a smoother one u connected with v by the elliptic system $v = u - \alpha^2 \Delta u$, has been constructed. The used operator is the well known Helmholtz operator. The

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