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FIXED POINTS AND THE INVERSE PROBLEM FOR CENTRAL CONFIGURATIONS

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Abstract. Central configurations play an important role in the dynamics of the n-body problem: they occur as relative equilibria and as asymptotic configurations in colliding trajectories. We illustrate how they can be found as projective fixed points of self-maps defined on the shape space, and some results on the inverse problem in dimension 1, i.e. finding (positive or real) masses which make a given collinear configuration central. This survey article introduces readers to the recent results of the author, also unpublished, showing an application of the fixed point theory.

1. Introduction

Let $n \geq 3$ be and integer, and $m_1, \ldots, m_n$ positive parameters, masses. Given a dimension $d \geq 2$, a configuration of n points in $\mathbb{R}^d$ is a n-tuple $\boldsymbol{q} = (\boldsymbol{q}_1, \ldots, \boldsymbol{q}_n)$ with $\boldsymbol{q}_j \in \mathbb{R}^d$ for all j and $\boldsymbol{q}_i \neq \boldsymbol{q}_j$ whenever $i \neq j$. Spaces of configurations (configuration spaces) have been the object of much of study in recent decades (see Fadell–Husseini [6] for a topological point of view and some deep and interesting consequences). The set of all configurations is denoted $\mathbb{F}_n(\mathbb{R}^d)$, following Fadell–Husseini notation. Topological properties of configuration spaces have consequences in the study of dynamical systems of n point particles interacting with mutual forces (the n-body problem: see for example [22]). One of the most

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