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SECTIONAL CATEGORY AND THE FIXED POINT PROPERTY

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Abstract. For a Hausdorff space X, we exhibit an unexpected connection between the sectional number of the Fadell–Neuwirth fibration $\pi_{2,1}^X\colon F(X,2)\to X$, and the fixed point property (FPP) for self-maps on X. Explicitly, we demonstrate that a space X has the FPP if and only if 2 is the minimal cardinality of open covers $\{U_i\}$ of X such that each U_i admits a continuous local section for $\pi_{2,1}^X$. This characterization connects a standard problem in fixed point theory to current research trends in topological robotics.

1. Introduction, outline and main results

A topological theory of motion planning was initiated in [6]. As a result, Farber's topological complexity of the space of states of an autonomous agent and, more generally, the sectional number of a map are numerical invariants appearing naturally in the emerging field of topological robotics (see [13] or [14]).

Let X be a topological space and $k\geq 1$. The ordered configuration space of k distinct points on X (see [5]) is the topological space

$$F(X,k)=\big\{(x_1,\dots,x_k)\in X^k \mid x_i\neq x_j \text{ whenever } i\neq j\big\},$$

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