

DYNAMICAL ZETA FUNCTIONS OF REIDEMEISTER TYPE

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ABSTRACT. In this paper we study dynamical representation theory zeta functions counting numbers of fixed irreducible representations for iterations of group endomorphism. The rationality and functional equation for these zeta functions are proven for several classes of groups. We prove Pólya–Carlson dichotomy between rationality and a natural boundary for analytic behavior of the Reidemeister zeta functions for a large class of automorphisms of infinitely generated Abelian groups. We also establish the connection between the Reidemeister zeta function and dynamical representation theory zeta functions under restriction of endomorphism to a subgroup.

1. Introduction

Let G be a countable discrete group and $\phi: G \rightarrow G$ an endomorphism. Two elements $\alpha, \beta \in G$ are said to be ϕ -conjugate or *twisted conjugate* if and only if there exists $g \in G$ with $\beta = g\alpha\phi(g^{-1})$. We shall write $\{x\}_\phi$ for the ϕ -conjugacy or *twisted conjugacy* class of the element $x \in G$. The number of ϕ -conjugacy classes is called the *Reidemeister number* of an endomorphism ϕ and is denoted by $R(\phi)$. If ϕ is the identity map then the ϕ -conjugacy classes are the usual conjugacy classes in the group G . Taking a dynamical point of view, we consider

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