

GENERALIZED LIMIT THEOREM AND BIFURCATION FOR PROBLEMS WITH PUCCI'S OPERATOR

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ABSTRACT. We establish a new limiting result which extends the famous Whyburn's limit theorem. As applications, we study the existence and multiplicity of one-sign or sign-changing solutions with a prescribed number of simple zeros for the following problem

$$\begin{cases} -\mathcal{M}_{\lambda,\Lambda}^+(D^2u) = \mu f(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\mathcal{M}_{\lambda,\Lambda}^+$ denotes the Pucci extremal operator. Combining bifurcation approach with our generalized limit theorem, we determine the range of parameter μ in which the above problem has one or multiple one-sign or sign-changing solutions according to the behaviors of f at 0 and ∞ , and whether f satisfies the signum condition $f(s)s > 0$ for $s \neq 0$.

1. Introduction and main results

Let X be a metric space with a metric ρ . We first recall the definition of ε -chain. If a and b are points, then by an ε -chain of points joining a and b is meant a finite sequence of points:

$$a = x_1, x_2, \dots, x_n = b$$

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