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L^∞-BOUNDS OF SOLUTIONS FOR A CLASS OF STRONGLY NONLINEAR ELLIPTIC EQUATIONS IN MUSIELAK SPACES

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Abstract. In this paper we establish the existence of bounded solutions to a strongly nonlinear elliptic problem of the form

$$-\operatorname{div}\mathcal{A}(x,u,\nabla u)+g(x,u,\nabla u)=f \quad \text{in } \Omega,$$

with $u\in W_0^1L_\varphi(\Omega)\cap L^\infty(\Omega)$, where

$$\mathcal{A}(x,s,\xi)\cdot\xi\geq\overline{\varphi}_x^{-1}(\varphi(x,h(|s|)))\varphi(x,|\xi|),$$

$h\colon\mathbb{R}^+\to\,]0,1]$ is a continuous decreasing function with unbounded primitive and g is a non-linearity satisfying $|g(x,s,\xi)|\leq\beta(s)\varphi(x,|\xi|)$. We assume the Δ_2-condition on the Musielak function φ.

1. Introduction

In the last decade, there has been an increasing interest in the study of various mathematical problems in modular spaces motivated by many considerations in applications, as the high number of papers and preprints recently appeared on the servers can witness (see for instance [20]–[22], [14], [13], …). Exponent Sobolev spaces, Orlicz–Sobolev spaces and variable exponent Sobolev spaces are special kinds of Musielak–Sobolev spaces which is defined by a more general function $\varphi(x,t)$ which may vary with the location in space.

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