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SEMIFLOW SELECTION AND MARKOV SELECTION THEOREMS

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Abstract. The deterministic analog of the Markov property of a time-homogeneous Markov process is the semigroup property of solutions of an autonomous differential equation. The semigroup property arises naturally when the solutions of a differential equation are unique, and leads to a semiflow. We prove an abstract result on the measurable selection of a semiflow for the situations without uniqueness. We outline applications to ODEs, PDEs, differential inclusions, etc. Our proof of the semiflow selection theorem is motivated by N.V. Krylov's Markov selection theorem. To accentuate this connection, we include a new version of the Markov selection theorem related to more recent papers of Flandoli & Romito and Goldys et al.

1. Introduction

This paper originates in our studies of the Markov property for statistical solutions of evolution PDEs. There, in a probabilistic setting, the fundamental result is the Markov selection theorem of N.V. Krylov [28] and its connection with the martingale problem as shown by D.W. Stroock and S.R.S. Varadhan [37]. While reading these sources along with more recent developments of F. Flandoli and M. Romito [15] and B. Goldys et al. [17], we have realized that there is a deterministic result lurking behind probabilistic considerations. The deterministic

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