Topological Methods in Nonlinear Analysis Volume 56, No. 1, 2020, 161–172 DOI: 10.12775/TMNA.2020.007

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IDEMPOTENT MEASURES: ABSOLUTE RETRACTS AND SOFT MAPS

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ABSTRACT. We investigate under which conditions the space of idempotent measures is an absolute retract and the idempotent barycenter map is soft.

1. Introduction

The notion of idempotent (Maslov) measure finds important applications in different part of mathematics, mathematical physics and economics (see the survey article [8] and the bibliography therein). Topological and categorical properties of the functor of idempotent measures were studied in [19]. Although idempotent measures are not additive and corresponding functionals are not linear, there are some parallels between topological properties of the functor of probability measures and the functor of idempotent measures (see for example [19] and [12]) which are based on existence of natural equiconnectedness structure on both functors.

However, some differences appeared when the problem of the openness of the barycenter map was studying. The problem of the openness of the barycenter map of probability measures was investigated in [4]–[6], [10] and [11]. In particular, it is proved in [10] that the barycenter map for a compact convex set in a locally convex space is open if and only if the map $(x, y) \mapsto 1/2(x + y)$ is open. Zarichnyi defined in [19] the idempotent barycenter map for idempotent

²⁰²⁰ Mathematics Subject Classification. 52A01, 54C10, 28A33, 54C55.

Key words and phrases. Absolute retract; soft map; idempotent (Maslov) measure; idempotent barycenter map.