

SEMICLASSICAL STATES FOR CRITICAL CHOQUARD EQUATIONS WITH CRITICAL FREQUENCY

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ABSTRACT. We study the multiplicity of semiclassical states for the Choquard equation

$$-\varepsilon^2 \Delta u + V(x)u = \varepsilon^{\mu-N} \left(\int_{\mathbb{R}^N} \frac{G(y, u(y))}{|x-y|^\mu} dy \right) g(x, u) \quad \text{in } \mathbb{R}^N,$$

where $0 < \mu < N$, $N \geq 3$, ε is a positive parameter and G is the primitive of g which is of critical growth due to the Hardy–Littlewood–Sobolev inequality. The potential function V is assumed to be nonnegative with $V(x) = 0$ in some region of \mathbb{R}^N . Using the genus theory we prove the multiplicity of semiclassical states for the critical Choquard equation.

1. Introduction and main results

The nonlinear Choquard equation

$$(1.1) \quad -\Delta u + V(x)u = (|x|^{-\mu} * |u|^q) |u|^{q-2} u \quad \text{in } \mathbb{R}^N,$$

arises in various areas of mathematical physics, e.g. in the quantum theory of a polaron at rest, Pekar [36], in modeling of an electron trapped in its own hole,

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