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KNOT INVARIANTS COMING FROM PRE-IMAGE INDICES

Xing Chen — Xuezhi Zhao

Abstract. Some sort of homomorphism indices of relative maps at pre-image are obtained. By using these indices, we define new invariants for knots, and prove their additivity.

1. Introduction

The Lefschetz fixed point theorem says that a self-map $f\colon X \to X$ on a compact polyhedron X must have a fixed point if its Lefschetz number $L(f)$ is non-zero. This theorem is regarded as one of the sources of the topological fixed point theory, because $L(f)$ is defined by the use homology (or cohomology), see [3]. A relative version of Lefschetz fixed point theorem was introduced by Bowszyc in [1]. It says that a relative self-map $f\colon (X, A) \to (X, A)$ has a fixed point on the closure $\overline{X - A}$ of the complement $X - A$ if the relative Lefschetz number $L(f) - L(f|_A)$ is non-zero.

The background of fixed point theory comes from the solvability of equations. Another more direct concept in this area is the degree, which was introduced by Hopf or the root theory which was developed by Brooks, see [2]. Recently, a more general roots has been considered: the so-called pre-images of a subset, see [4]. The key point is the definition of homomorphism indices. Some ideas behind the relations among of the roots, fixed points and pre-images can be found in [6].

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