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CRITICAL NEUMANN PROBLEMS WITH ASYMMETRIC NONLINEARITY

Francisco Odair de Paiva — Wallisom Rosa

Abstract. We prove an existence result for a semilinear elliptic equation with superlinear and asymmetric nonlinearity. The asymmetry that we consider is of the type: linear at $-\infty$ and superlinear at $+\infty$. To obtain these results we apply a Linking Theorem.

1. Introduction

Consider the semilinear elliptic problem

$$\begin{cases} -\Delta u = \lambda u + g(x,u) + (u^+)^{2^*-1} & \text{for } x \in \Omega, \\ \dfrac{\partial u}{\partial \nu} = 0 & \text{for } x \in \partial\Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 3$, is a bounded domain with smooth boundary $\partial\Omega$, $\lambda > 0$ and $2^* = 2N/(N-2)$ is the critical Sobolev exponent. Moreover, it is assumed that $g\colon \overline{\Omega} \times \mathbb{R} \to \mathbb{R}$ is continuous and satisfies

(g1) $g(x,s) = 0$ if $s \leq 0$ and $g(x,s) > 0$ if $s > 0$;

(g2) there exist $\sigma \in (1, 2^*-1)$ and a constant $K > 0$ such that

$$|g(x,s)| \leq K|s|^\sigma, \quad \text{for all } x \in \Omega \text{ and for all } s \in \mathbb{R};$$

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