

SPECTRAL DECOMPOSITION AND STABILITY OF MILD EXPANSIVE SYSTEMS

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ABSTRACT. In this paper we introduce another type of expansiveness, called mild expansiveness, for homeomorphisms on compact metric spaces, and present a topological version of the spectral decomposition theorem by Smale [17] and Bowen [5]. Moreover we study the topological stability of mild expansive homeomorphisms, and characterize the mild expansive diffeomorphisms on compact smooth manifolds using the notion of Ω -stability.

1. Introduction

The idea of expansivity in dynamical systems was introduced in the middle of the twentieth century, and has played an important role in the qualitative study of dynamical systems. Roughly speaking, a system is expansive if two orbits cannot remain close to each other under the action of the system. For a homeomorphism f of a compact metric space (X, d) , we say that f is *expansive* if there is $\delta > 0$ such that $\Gamma_\delta(x) = \{x\}$ for all $x \in X$, where

$$\Gamma_\delta(x) = \{y \in X : d(f^n(x), f^n(y)) \leq \delta \text{ for all } n \in \mathbb{Z}\}.$$

In the light of the rich consequence of expansiveness in the dynamics of a system, it is natural to consider another notions of expansiveness like N -expansiveness,

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