

**LOWER SEMICONTINUITY
OF THE PULLBACK ATTRACTORS
OF NON-AUTONOMOUS DAMPED WAVE EQUATIONS
WITH TERMS CONCENTRATING ON THE BOUNDARY**

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ABSTRACT. In this paper we analyze the asymptotic behavior of the pullback attractors for non-autonomous dynamical systems generated by a family of non-autonomous damped wave equations when some reaction terms are concentrated in a neighbourhood of the boundary and this neighbourhood shrinks to boundary as a parameter ε goes to zero. We show the gradient-like structure of the limit pullback attractor, the existence and continuity of global hyperbolic solutions and the lower semicontinuity of the pullback attractors at $\varepsilon = 0$. Finally, we obtain the continuity of the pullback attractors at $\varepsilon = 0$.

1. Introduction

In this work, we analyze the asymptotic behavior of the pullback attractors of a non-autonomous damped wave equation when some reaction terms are concentrated in a neighbourhood of the boundary and this neighbourhood shrinks to boundary as a parameter ε goes to zero. This work completes the analysis made in our article Aragão and Bezerra [1] and [2].

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Key words and phrases. Wave equation; non-autonomous; concentrating terms; pullback attractor; gradient-like; hyperbolic solution; lower semicontinuity.

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