

# MULTIPLE SOLUTIONS FOR QUASILINEAR EQUATION INVOLVING HARDY CRITICAL SOBOLEV EXPONENTS

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ABSTRACT. We consider the following quasilinear elliptic equation with critical Sobolev and Hardy–Sobolev exponents:

$$\begin{cases} -\sum_{i,j=1}^N D_j(b_{ij}(v)D_i v) + \frac{1}{2} \sum_{i,j=1}^N b'_{ij}(v)D_i v D_j v \\ \quad = \frac{|v|^{2_s^* q-2} v}{|x|^s} + \mu |v|^{2^* q-2} v + a(x)|v|^{2q-2} v & \text{in } \Omega, \\ v = 0 & \text{on } \partial\Omega, \end{cases}$$

where  $b_{ij} \in C^1(\mathbb{R}, \mathbb{R})$  satisfies the growth condition  $|b_{ij}(t)| \sim |t|^{2(q-1)}$  at infinity,  $q \geq 1$ ,  $\mu \geq 0$ ,  $0 < s < 2$ ,  $2_s^* = 2(N-s)/(N-2)$ ,  $2^* = 2N/(N-2)$ ,  $0 \in \bar{\Omega}$  and  $\Omega$  is a bounded domain in  $\mathbb{R}^N$ . In this paper, we will investigate the effects of the lower order terms  $a(x)|v|^{2q-2}v$  and the growth of  $b_{ij}(v)$  at infinity on the existence of infinitely many solutions for the above equations.

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