MULTIPLE SOLUTIONS
FOR QUASILINEAR EQUATION
INVOLVING HARDY CRITICAL SOBOLEV EXPONENTS

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ABSTRACT. We consider the following quasilinear elliptic equation with critical Sobolev and Hardy–Sobolev exponents:

$$
\begin{cases}
-\displaystyle\sum_{i,j=1}^{N} D_j(b_{ij}(v)D_i v) + \frac{1}{2}\sum_{i,j=1}^{N} b'_{ij}(v)D_i v D_j v \\
\qquad = \dfrac{|v|^{2^*_s q-2}v}{|x|^s} + \mu|v|^{2^* q-2}v + a(x)|v|^{2q-2}v \quad \text{in } \Omega, \\
v = 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \text{on } \partial\Omega,
\end{cases}
$$

where $b_{ij} \in C^1(\mathbb{R}, \mathbb{R})$ satisfies the growth condition $|b_{ij}(t)| \sim |t|^{2(q-1)}$ at infinity, $q \geq 1$, $\mu \geq 0$, $0 < s < 2$, $2^*_s = 2(N-s)/(N-2)$, $2^* = 2N/(N-2)$, $0 \in \overline{\Omega}$ and Ω is a bounded domain in $\mathbb{R}^N$. In this paper, we will investigate the effects of the lower order terms $a(x)|v|^{2q-2}v$ and the growth of $b_{ij}(v)$ at infinity on the existence of infinitely many solutions for the above equations.