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# ON THE BOUNDED INDEX PROPERTY FOR PRODUCTS OF ASPHERICAL POLYHEDRA

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Abstract. A compact polyhedron $X$ is said to have the Bounded Index Property for Homotopy Equivalences (BIPHE) if there is a finite bound $\mathcal{B}$ such that for any homotopy equivalence $f\colon X \to X$ and any fixed point class $\mathbf{F}$ of $f$, the index $|\mathrm{ind}(f, \mathbf{F})| \leq \mathcal{B}$. In this note, we consider the product of compact polyhedra, and give some sufficient conditions for it to have BIPHE. Moreover, we show that products of closed Riemannian manifolds with negative sectional curvature, in particular hyperbolic manifolds, have BIPHE, which gives an affirmative answer to a special case of a question asked by Boju Jiang.

## 1. Introduction

In the fixed point theory, fixed points of a self-map $f$ of a space $X$ are studied. The Nielsen fixed point theory, in particular, is concerned with the properties of the fixed point set

$$\mathrm{Fix}\, f := \{x \in X \mid f(x) = x\}$$

that are invariant under homotopy of a map $f$ (see [2] for an introduction).

The fixed point set $\mathrm{Fix} f$ splits into a disjoint union of *fixed point classes*: two fixed points $a$ and $a'$ are in the same class if and only if there is a lifting $\widetilde{f}\colon \widetilde{X} \to \widetilde{X}$ of $f$ such that $a, a' \in p(\mathrm{Fix}\, \widetilde{f})$, where $p\colon \widetilde{X} \to X$ is the universal

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