

SEMIGROUPS ON TIME SCALES AND APPLICATIONS TO ABSTRACT CAUCHY PROBLEMS

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ABSTRACT. In this paper, we introduce the definition of a C_0 -semigroup on a time scale, which unifies the continuous, discrete and other cases which lie between them. Also, it extends the classical theory of operator semigroups to the quantum case. We study the relationship between the semigroup and its infinitesimal generator. We apply our theory to study the homogeneous and non homogeneous abstract Cauchy problem in Banach and Fréchet spaces.

1. Introduction

The theory of time scales was introduced in the literature by Stefan Hilger in 1988. Since then, this theory has been extensively investigated by several researchers, specially due to its applications in different fields of knowledge such as economics, physics, biology, engineer, chemistry, among others. See, for instance, [1], [4], [8]–[10], [14], [18], [19], [22], [26] and the references therein.

Since a time scale $\mathbb{T}$ is any closed nonempty subset of $\mathbb{R}$, this theory allows to unify several existing concepts, depending on the chosen time scale. For instance,

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