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## TOPOLOGICAL OPTIMIZATION VIA COST PENALIZATION

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Dedicated to the memory of Prof. Dr. Ioan I. Vrabie

ABSTRACT. We consider general shape optimization problems governed by Dirichlet boundary value problems. The proposed approach may be extended to other boundary conditions as well. It is based on a recent representation result for implicitly defined manifolds, due to the authors, and it is formulated as an optimal control problem. The discretized approximating problem is introduced and we give an explicit construction of the associated discrete gradient. Some numerical examples are also indicated.

## 1. Introduction

Shape optimization is a relatively young branch of mathematics, with important modern applications in engineering and design. Certain optimization problems in mechanics, thickness optimization for plate or rods, geometric optimization of shells, curved rods, drag minimization in fluid mechanics, etc. are some examples. Many appear naturally in the form of control by coefficients problems, due to the formulation of the mechanical models, with the geometric characteristics entering the coefficients of the differential operators. See [15, Chapter 6], where such questions are discussed in details.

It is the aim of this article to develop an optimal control approach, using penalization methods, to general shape optimization problems as investigated in [20], [23], [5], [10], [8], etc. We underline that our methodology allows simultaneous topological and boundary variations.

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