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FRACTIONAL HARDY–SOBOLEV ELLIPTIC PROBLEMS

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Abstract. In this paper, we study the following singular nonlinear elliptic problem

$$\text{(P)} \qquad \begin{cases} (-\Delta)^{\alpha/2}u = \lambda|u|^{r-2}u + \mu\,\dfrac{|u|^{q-2}u}{|x|^s} & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a smooth bounded domain in $\mathbb{R}^N (N \geq 2)$ with $0 \in \Omega$, $\lambda, \mu > 0$, $0 < s \leq \alpha$, $(-\Delta)^{\alpha/2}$ is the spectral fractional Laplacian operator with $0 < \alpha < 2$. We establish existence results and nonexistence results of problem (P) for subcritical, Sobolev critical and Hardy–Sobolev critical cases.

1. Introduction

The main objective of this paper is to consider the following fractional Hardy–Sobolev elliptic problem:

$$\begin{cases} (-\Delta)^{\alpha/2}u = \lambda|u|^{r-2}u + \mu\,\dfrac{|u|^{q-2}u}{|x|^s} & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where Ω is a smooth bounded domain in $\mathbb{R}^N$ $(N \geq 2)$ with $0 \in \Omega$, $\lambda, \mu > 0$, $0 < s \leq \alpha$. $(-\Delta)^{\alpha/2}$ $(0 < \alpha < 2)$ is the spectral fractional Laplacian operator which is defined as follows.

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