Topological **M**ethods in **N**onlinear **A**nalysis
Volume 55, No. 1, 2020, 243–255
DOI: 10.12775/TMNA.2019.074

POSITIVE SOLUTIONS OF SEMIPOSITONE ELLIPTIC PROBLEMS WITH CRITICAL TRUDINGER–MOSER NONLINEARITIES

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Abstract. We prove the existence of a positive solution to a semipositone N-Laplacian problem with a critical Trudinger–Moser nonlinearity. The proof is based on obtaining uniform $C^{1,\alpha}$ a priori estimates via a compactness argument. Our result is new even in the semilinear case $N = 2$, and our arguments can easily be adapted to obtain positive solutions of more general semipositone problems with critical Trudinger–Moser nonlinearities.

1. Introduction

Elliptic problems with critical Trudinger–Moser nonlinearities have been widely investigated in the literature. We refer the reader to the survey paper of de Figueiredo et al. [2] for an overview of recent results on Trudinger–Moser type inequalities and related critical problems. A model critical problem of this type is

$$\begin{cases} -\Delta_N u = \lambda |u|^{N-2} u \, e^{\beta |u|^{N'}} & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

2020 *Mathematics Subject Classification.* Primary: 35J92; Secondary: 35B33, 35B09, 35B45.

Key words and phrases. Semipositone N-Laplacian problems; critical Trudinger–Moser nonlinearities; positive solutions; uniform $C^{1,\alpha}$ a priori estimates.

The second author was supported by the National Research Foundation of Korea, grant funded by the Korea Government (MEST) (NRF2018R1D1A3A03000678).