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## HÖLDER BEHAVIOR OF VISCOSITY SOLUTIONS OF SOME FULLY NONLINEAR EQUATIONS IN THE HEISENBERG GROUP

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ABSTRACT. In this paper we prove the  $C^{0,\alpha}$  regularity of bounded and uniformly continuous viscosity solutions of some degenerate fully nonlinear equations in the first Heisenberg group.

## 1. Introduction

In this paper we prove  $C^{0,\alpha}$  regularity of bounded and uniformly continuous viscosity solutions of some degenerate fully nonlinear elliptic equations in the first Heisenberg group  $\mathbb{H}$ . It is known that the theory of viscosity solutions is very flexible and that the existence of viscosity solutions of second order PDEs is not strictly related to the degeneracy of the elliptic operator, see [15], [13]. The regularity of viscosity solutions of second order elliptic, possibly nonlinear, PDEs is also well established: one of the key ingredients is given by the Harnack inequality which, in turn, is based on the Alexandroff–Bakelman–Pucci inequality, ABP in short, and the consequent maximum principle. We refer to the

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