Topological Methods in Nonlinear Analysis Volume 54, No. 2A, 2019, 459–476 DOI: 10.12775/TMNA.2019.045

© 2019 Juliusz Schauder Centre for Nonlinear Studies Nicolaus Copernicus University in Toruń

## SOME TWO-POINT PROBLEMS FOR SECOND ORDER INTEGRO-DIFFERENTIAL EQUATIONS WITH ARGUMENT DEVIATIONS

Sulkhan Mukhigulashvili — Veronika Novotná

ABSTRACT. In the paper we describe the classes of unique solvability of the Dirichlet and mixed two point boundary value problems for the second order linear integro-differential equation

 $u''(t) = p_0(t)u(t) + p_1(t)u(\tau_1(t)) + \int_a^b p(t, s)u(\tau(s)) \, ds + q(t).$ 

On the basis of the obtained and, in some sense, optimal results for the linear problems, by the a priori boundedness principle we prove the theorems of solvability and unique solvability for the second order nonlinear functional differential equations under the mentioned boundary conditions.

## 1. Statement of the main results

**1.1. Introduction.** In this paper we will consider the second order linear integro-differential equation

(1.1) 
$$u''(t) = p_0(t)u(t) + p_1(t)u(\tau_1(t)) + \int_a^b p(t,s)u(\tau(s))\,ds + q(t)$$

<sup>2010</sup> Mathematics Subject Classification. Primary: 34K06, 34K10; Secondary: 34B15. Key words and phrases. Integro-differential equations; Dirichlet and mixed problems; unique solvability; a priori boundedness principle.

The final version of this paper will be submitted for publication elsewhere.

For S. Mukhigulashvili the research was supported by institutional grant RVO: 67985840. For V. Novotná research was supported by the Czech Science Foundation: GA16-03796S.