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EXISTENCE, LOCALIZATION AND STABILITY OF LIMIT-PERIODIC SOLUTIONS TO DIFFERENTIAL EQUATIONS INVOLVING CUBIC NONLINEARITIES

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Dedicated to the memory of Professor Ioan I. Vrabie

Abstract. We will prove, besides other things like localization and (in)stability, that the differential equations $x' + x^3 - \lambda x = \varepsilon r(t)$, $\lambda > 0$, and $x'' + x^3 - x = \varepsilon r(t)$, where $r\colon \mathbb{R} \to \mathbb{R}$ are uniformly limit-periodic functions, possess for sufficiently small values of $\varepsilon > 0$ uniformly limit-periodic solutions, provided r in the first-order equation is strictly positive. As far as we know, these are the first nontrivial effective criteria, obtained for limit-periodic solutions of nonlinear differential equations, in the lack of global lipschitzianity restrictions. A simple illustrative example is also indicated for difference equations.

1. Introduction

As our title indicates, the main aim of the present paper is to study limit-periodic solutions of the first-order and the second-order differential equations involving cubic nonlinearities and limit-periodic forcing terms. The investigation of limit-periodic nonlinear oscillations is a delicate problem, especially because the space of limit-periodic functions endowed with the sup-norm is complete, but not linear. That is why the related results are, unlike those for periodic

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