

FORMAL BARYCENTER SPACES WITH WEIGHTS: THE EULER CHARACTERISTIC

SADOK KALLEL

ABSTRACT. We compute the Euler characteristic with compact supports χ_c of the formal barycenter spaces with weights of some locally compact spaces, connected or not. This reduces to the topological Euler characteristic χ when the weights of the singular points are less than one. As foresighted by Andrea Malchiodi, our formula is related to the Leray–Schauder degree for mean field equations on a compact Riemann surface obtained by C.C. Chen and C.S. Lin.

1. Statement of the main result

Given a space X , we will write $\mathcal{B}_k(X)$ for the space of formal barycenters of k points in X [11]. By construction there are inclusions $\mathcal{B}_k(X) \hookrightarrow \mathcal{B}_{k+1}(X)$ for all k and we will write $\mathcal{B}(X)$ the direct limit. This is known to be a contractible space if X is of the homotopy type of a CW.

Let $Q_r := \{y_1, \dots, y_r\} \subset X$ be a fixed finite set of “singular points” in X . We assign to every $x \in X$ a *weight*

$$w(x) = \begin{cases} 1 & \text{if } x \notin Q_r, \\ w_i & \text{if } x = y_i, \end{cases}$$

2010 *Mathematics Subject Classification.* 55M99, 57N80.

Key words and phrases. Euler characteristic; compact supports; Leray–Schauder degree; stratification.