Topological **M**ethods in **N**onlinear **A**nalysis
Volume 53, No. 2, 2019, 603–621
DOI: 10.12775/TMNA.2019.013

MULTIPLICITY RESULTS FOR FRACTIONAL p-LAPLACIAN PROBLEMS WITH HARDY TERM AND HARDY–SOBOLEV CRITICAL EXPONENT IN $\mathbb{R}^N$

Hadi Mirzaee

Abstract. This paper is devoted to the study of a class of singular fractional p-Laplacian problems of the form

$$(-\Delta)_p^s u - \mu\,\frac{|u|^{p-2}u}{|x|^{ps}} = \alpha\,\frac{|u|^{p_s^*(b)-2}u}{|x|^b} + \beta f(x)|u|^{q-2}u \quad \text{in } \mathbb{R}^N$$

where $0 < s < 1$, $0 \le b < ps < N$, $1 < q < p_s^*(b)$, $\alpha, \beta > 0$, $\mu \in \mathbb{R}$, and $f(x)$ is a given function which satisfies some appropriate condition. By using variational methods, we prove the existence of infinitely many solutions under different conditions.

1. Introduction and statement of main result

In this article, we consider the following fractional p-Laplacian equations with Hardy term and Hardy–Sobobev critical exponent:

$$(-\Delta)_p^s u - \mu\,\frac{|u|^{p-2}u}{|x|^{ps}} = \alpha\,\frac{|u|^{p_s^*(b)-2}u}{|x|^b} + \beta f(x)|u|^{q-2}u \quad \text{in } \mathbb{R}^N \tag{1.1}$$

where $0 < s < 1$, $0 \le b < ps < N$, $1 < q < p_s^*(b) = p(N-b)/(N-ps)$, $\alpha, \beta > 0$ and $\mu \in \mathbb{R}$. The operator $(-\Delta)_p^s$ is the fractional p-Laplacian, which up to

2010 *Mathematics Subject Classification.* 35J60, 35R11, 35J20.
Key words and phrases. Hardy term; fractional p-Laplacian; critical exponent.