Topological Methods in Nonlinear Analysis
Volume 53, No. 2, 2019, 575–601
DOI: 10.12775/TMNA.2019.012

REIDEMEISTER SPECTRA FOR SOLVMANIFOLDS IN LOW DIMENSIONS

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Abstract. The Reidemeister number of an endomorphism of a group is the number of twisted conjugacy classes determined by that endomorphism. The collection of all Reidemeister numbers of all automorphisms of a group G is called the Reidemeister spectrum of G. In this paper, we determine the Reidemeister spectra of all fundamental groups of solvmanifolds up to Hirsch length 4.

1. Introduction

Let G be a group and $\varphi\colon G \to G$ an endomorphism. Consider the following equivalence relation on G:

$$x \sim_\varphi y \quad \text{if and only if} \quad \exists\, z \in G : x = zy\varphi(z)^{-1}.$$

The equivalence classes under $\sim_\varphi$ are the *Reidemeister classes of* φ or the *φ-twisted conjugacy classes*. We denote the set of these equivalence classes by $\mathcal{R}(\varphi)$. The number of equivalence classes is called the *Reidemeister number of* φ and is denoted by $R(\varphi)$. If $\mathcal{R}(\varphi)$ is infinite, we write $R(\varphi) = \infty$. Subsequently, the *Reidemeister spectrum of* G is defined as

$$\mathrm{Spec}_{\mathrm{R}}(G) := \{R(\varphi) \mid \varphi \in \mathrm{Aut}(G)\}.$$

2010 *Mathematics Subject Classification.* 20E36, 20F16.
Key words and phrases. Twisted conjugacy; Reidemeister number; polycyclic group; solvmanifold.