

## AMENABILITY AND HAHN–BANACH EXTENSION PROPERTY FOR SET VALUED MAPPINGS

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**ABSTRACT.** Amenability is an important notion in harmonic analysis on groups and semigroups, and their associated Banach algebras. In this paper, we present some characterization of a semitopological semigroup  $S$  on the existence of a left invariant mean on  $LUC(S)$ ,  $AP(S)$  and  $WAP(S)$  in terms of the Hahn–Banach extension theorem, which extend the first author’s early results in 1970s. Moreover, we refine and extend the well known Day’s result and Mitchell’s results on fixed point properties for set-valued mappings. As an application, we give an application of our result to a class of Banach algebras related to amenability of groups and semigroups.

### 1. Introduction

Throughout this paper, we assume that  $E$  is a real separated locally convex space. All topologies in this paper are assumed to be Hausdorff.

Let  $A: E \rightrightarrows E$  be a *set-valued operator* (also known as multifunction) from  $E$  to  $E$ , i.e. for every  $x \in E$ ,  $Ax \subseteq E$ , and let  $\text{gra } A := \{(x, y) \in E \times E \mid y \in Ax\}$  be the *graph* of  $A$ ,  $\text{dom } A := \{x \in E \mid Ax \neq \emptyset\}$  be the *domain* of  $A$ . We say that  $A$  is a *linear relation* if  $\text{gra } A$  is a subspace of  $E \times E$ .

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