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ABOUT POSITIVE $W^{1,\Phi}_{\mathrm{loc}}(\Omega)$-SOLUTIONS TO QUASILINEAR ELLIPTIC PROBLEMS WITH SINGULAR SEMILINEAR TERM

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Abstract. This paper deals with the existence, uniqueness and regularity of positive $W^{1,\Phi}_{\mathrm{loc}}(\Omega)$-solutions of singular elliptic problems on a smooth bounded domain with Dirichlet boundary conditions involving the Φ-Laplacian operator. The proof of the existence is based on a variant of the generalized Galerkin method that we developed inspired by ideas of Browder [4] and a comparison principle. By the use of a kind of Moser's iteration scheme we show the $L^\infty(\Omega)$-regularity for positive solutions.

1. Introduction

The paper concerns the existence, uniqueness and regularity of $W^{1,\Phi}_{\mathrm{loc}}(\Omega)$-solutions to the singular elliptic problem

$$(1.1)\qquad -\mathrm{div}(\phi(|\nabla u|)\nabla u) = \frac{a(x)}{u^\alpha} \quad \text{in } \Omega, \qquad u>0 \quad \text{in } \Omega, \qquad u=0 \quad \text{on } \partial\Omega,$$

where $\Omega \subset \mathbb{R}^N$, with $N \geq 2$, is a bounded domain with smooth boundary $\partial\Omega$, a is a non-negative function, $0<\alpha<\infty$ and $\phi\colon (0,\infty) \to (0,\infty)$ is of class C^1 and satisfies

(ϕ_1) (i) $t\phi(t) \to 0$ as $t \to 0$,

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