

## EXTREME PARTITIONS OF A LEBESGUE SPACE AND THEIR APPLICATION IN TOPOLOGICAL DYNAMICS

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**ABSTRACT.** It is shown that any topological action  $\Phi$  of a countable orderable and amenable group  $G$  on a compact metric space  $X$  and every  $\Phi$ -invariant probability Borel measure  $\mu$  admit an extreme partition  $\zeta$  of  $X$  such that the equivalence relation  $R_\zeta$  associated with  $\zeta$  contains the asymptotic relation  $A(\Phi)$  of  $\Phi$ . As an application of this result and the generalized Glasner theorem it is proved that  $A(\Phi)$  is dense for the set  $E_\mu(\Phi)$  of entropy pairs.

### 1. Introduction

In the paper we consider topological dynamical systems on a compact metric space being actions of a countable amenable and orderable (CAO) group.

The simplest class of CAO groups applied in topological dynamics is formed by the groups  $\mathbb{Z}^d$ ,  $d \geq 1$ . One can show that all finitely generated, torsion-free nilpotent groups are CAO ([3], [7]).

In our further considerations we shall assume that the given compact metric space is equipped with a Borel measure invariant with respect to the considered action. Measurable partitions of the space form a useful tool in the theory of dynamical systems.

In particular extreme partitions play an important role in the entropy theory. The existence of extreme partitions for  $\mathbb{Z}^d$ -actions has been proved by Rokhlin

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