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# LIPSCHITZ RETRACTIONS ONTO SPHERE VS SPHERICAL CUP IN A HILBERT SPACE

Jumpot Intrakul — Phichet Chaoha — Wacharin Wichiramala

Abstract. We prove that, in every infinite dimensional Hilbert space, there exists $t_0 > -1$ such that the smallest Lipscthiz constant of retractions from the unit ball onto its boundary is the same as the smallest Lipschitz constant of retractions from the unit ball onto its $t$-spherical cup for all $t \in [-1, t_0]$.

## 1. Introduction and preliminaries

For a given Banach space $X$, it is known that $X$ has infinite dimension if and only if there exists a lipschitzian retraction from the closed unit ball $B_X$ onto its boundary (the unit sphere) $S_X$ (see [10] and [3]). Motivated by this fact, it is natural to ask for the value of the smallest Lipschitz constant of such a retraction, which is defined to be

$$k_0(X) := \inf\{k : \text{there exists a } k\text{-lipschitzian retraction from } B_X \text{ onto } S_X\}.$$

This question is generally regarded as the optimal retraction problem from the unit ball onto its sphere. Although the exact value of $k_0(X)$ has not been found even for one space $X$, some approximations are discovered. For example, see Table 1.

For the case of a Hilbert space $H$, the development of upper bounds of $k_0(H)$ in [8], [9], [4] and [2] is all based on the direct, yet very technical, constructions

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