

NONLINEAR UNILATERAL PARABOLIC PROBLEMS IN MUSIELAK–ORLICZ SPACES WITH L^1 DATA

MUSTAFA AIT KHELLOU — SIDI MOHAMED DOURI — YOUSSEF EL HADFI

ABSTRACT. We study, in Musielak–Orlicz spaces, the existence of solutions for some strongly nonlinear parabolic unilateral problem with L^1 data and without sign condition on nonlinearity.

1. Introduction

Let Ω be a bounded Lipschitz domain of \mathbb{R}^N ($N \geq 2$) and let $Q = \Omega \times (0, T)$, $T > 0$. Consider the following nonlinear parabolic problem:

$$(1.1) \quad \begin{cases} \frac{\partial u}{\partial t} + A(u) + g(x, t, u, \nabla u) = f & \text{in } Q, \\ u = 0 & \text{on } \partial\Omega \times (0, T), \\ u(x, 0) = u_0(x) & \text{in } \Omega, \end{cases}$$

where $A(u) = -\operatorname{div} a(x, t, u, \nabla u)$ is a Leray–Lions operator defined on $D(A) \subset W_0^{1,x} L_\varphi(Q) \rightarrow W^{-1,x} L_{\bar{\varphi}}(Q)$ with φ and $\bar{\varphi}$ two complementary Musielak–Orlicz functions, and g is a nonlinearity satisfying the growth condition

$$|g(x, t, s, \xi)| \leq c'(x, t) + b(s)\varphi(x, |\xi|),$$

where $b: \mathbb{R} \rightarrow \mathbb{R}^+$ is a continuous nondecreasing function in $L^1(\mathbb{R})$ and $c'(\cdot, \cdot)$ is a given nonnegative function in $L^1(Q)$.

2010 *Mathematics Subject Classification.* 35K55, 35K86, 46E30.

Key words and phrases. Musielak–Orlicz spaces; nonlinear unilateral parabolic problems; entropy solutions.