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EXISTENCE OF THREE NONTRIVIAL SOLUTIONS FOR A CLASS OF FOURTH-ORDER ELLIPTIC EQUATIONS

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Abstract. The existence of three nontrivial solutions is established for a class of fourth-order elliptic equations. Our technical approach is based on Linking Theorem and (∇)-Theorem.

1. Introduction and main results

We consider the fourth-order elliptic equation

$$\begin{cases} \Delta^2 u + c\Delta u = \mu u + f(x,u) & \text{in } \Omega, \\ u = \Delta u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^N$ $(N > 4)$ is a bounded smooth domain, $c \in \mathbb{R}$ and $f\colon \Omega \times \mathbb{R} \to \mathbb{R}$. Δ is the Laplace operator and Δ^2 is the biharmonic operator.

Let $0 < \lambda_1 < \ldots < \lambda_k < \ldots$ be the distinct eigenvalues of $-\Delta$ in $H_0^1(\Omega)$. The eigenvalue problem

$$\begin{cases} \Delta^2 u + c\Delta u = \mu u & \text{in } \Omega, \\ u = \Delta u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.2}$$

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