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EXISTENCE AND CONCENTRATION OF GROUND STATE SIGN-CHANGING SOLUTIONS FOR KIRCHHOFF TYPE EQUATIONS WITH STEEP POTENTIAL WELL AND NONLINEARITY

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Abstract. We study the following class of elliptic equations:

$$-\left(a+b\int_{\mathbb{R}^3}|\nabla u|^2\,dx\right)\Delta u+\lambda V(x)u=f(u),\quad x\in\mathbb{R}^3,$$

where $\lambda, a, b>0$, $V\in\mathcal{C}(\mathbb{R}^3,\mathbb{R})$ and $V^{-1}(0)$ has nonempty interior. First, we obtain one ground state sign-changing solution $u_{b,\lambda}$ applying the non-Nehari manifold method. We show that the energy of $u_{b,\lambda}$ is strictly larger than twice that of the ground state solutions of Nehari-type. Next we establish the convergence property of $u_{b,\lambda}$ as $b\searrow 0$. Finally, the concentration of $u_{b,\lambda}$ is explored on the set $V^{-1}(0)$ as $\lambda\to\infty$.

1. Introduction and preliminaries

In this paper, we are concerned with the following elliptic equations:

$$-\left(a+b\int_{\mathbb{R}^3}|\nabla u|^2\,dx\right)\Delta u+\lambda V(x)u=f(u),\quad x\in\mathbb{R}^3, \tag{1.1}$$

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Key words and phrases. Kirchhoff type equation; ground state sign-changing solutions; non-Nehari manifold method; concentration of solutions.

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