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NONZERO POSITIVE SOLUTIONS OF A MULTI-PARAMETER ELLIPTIC SYSTEM WITH FUNCTIONAL BCS

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This paper is dedicated to the memory of the late Panagiotis K. Palamides, for his teachings and friendship

ABSTRACT. We prove, by topological methods, new results on the existence of nonzero positive weak solutions for a class of multi-parameter second order elliptic systems subject to functional boundary conditions. The setting is fairly general and covers the case of multi-point, integral and nonlinear boundary conditions. We also present a non-existence result. We provide some examples to illustrate the applicability of our theoretical results.

1. Introduction

In this paper we discuss the solvability of the multi-parameter system of second order elliptic equations subject to functional boundary conditions (BCs)

(1.1)
$$\begin{cases} L_i u_i(x) = \lambda_i f_i(x, u(x)), & x \in \Omega, \quad i = 1, \dots, n, \\ B_i u_i(x) = \eta_i h_i[u], & x \in \partial \Omega, \quad i = 1, \dots, n, \end{cases}$$

where $\Omega \subset \mathbb{R}^m$ $(m \geq 2)$ is a bounded domain with sufficiently regular boundary, L_i is a strongly uniformly elliptic operator, B_i is a first order boundary operator,

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