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SCHRÖDINGER-POISSON SYSTEMS WITH RADIAL POTENTIALS AND DISCONTINUOUS QUASILINEAR NONLINEARITY

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Abstract. We consider the following Schrödinger–Poisson system:

$$\begin{cases} -\triangle u + V(|x|)u + \phi u = Q(|x|)f(u) & \text{in } \mathbb{R}^3, \\ -\triangle \phi = u^2 & \text{in } \mathbb{R}^3, \end{cases}$$

with more general radial potentials V,Q and discontinuous nonlinearity f. The Lagrange functional may be locally Lipschitz. Using nonsmooth critical point theorem, we obtain the multiplicity results of radial solutions, we also show concentration properties of the solutions. This is in contrast with some recent papers concerning similar problems by using the classical Sobolev embedding theorems.

1. Introduction and main results

In this paper we look for radial solutions of the following Schrödinger–Poisson system:

(1.1)
$$\begin{cases} -\triangle u + V(|x|)u + \phi u = Q(|x|)f(u) & \text{in } \mathbb{R}^3, \\ -\triangle \phi = u^2 & \text{in } \mathbb{R}^3. \end{cases}$$

Throughout the paper we assume V and Q are continuous, nonnegative functions in $(0, \infty)$. System (1.1) is also called the Schrödinger–Maxwell equation. Systems

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