Topological Methods in Nonlinear Analysis Volume 50, No. 2, 2017, 643–667 DOI: 10.12775/TMNA.2017.034

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## MAYER-VIETORIS PROPERTY OF THE FIXED POINT INDEX

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ABSTRACT. We study a Mayer–Vietoris kind formula for the fixed point index of maps of ENR triplets  $f\colon (X;X_1,X_2)\to (X;X_1,X_2)$  having compact fixed point set. We prove it under some suitable conditions. For instance when  $(X;X_1,X_2)=(E^n;E^n_+,E^n_-)$ .

We use these results to generalize the Poincaré–Bendixson index formula for vector fields to continuous maps having a sectorial decomposition, to study the fixed point index i(f,0) of orientation preserving homeomorphisms of  $E_+^2$  and  $(E^3; E_+^3, E_-^3)$  and the fixed point index in the invariant subspace.

## 1. Introduction

In this paper we will deal with triplets  $(X; X_1, X_2)$  where X is an ENR and  $X_1, X_2$  are ENR's closed in X such that  $X_1 \cap X_2$  is also an ENR and  $X = X_1 \cup X_2$ . We will call such triplets  $(X; X_1, X_2)$  ENR triplets and we will denote  $X_0 := X_1 \cap X_2$ . A continuous map  $f: (X; X_1, X_2) \to (X; X_1, X_2)$  of a triplet is a continuous map  $f: X \to X$  satisfying that

$$f(X_i) \subset X_i, \quad i = 1, 2.$$

Notice that  $f(X_0) \subset X_0$ .

We denote

$$f_i := f|_{X_i} \colon X_i \to X_i, \quad i = 0, 1, 2.$$

<sup>2010</sup> Mathematics Subject Classification. Primary: 37C25; Secondary: 37B30, 55M25. Key words and phrases. Fixed point index; Brouwer degree; sectorial decomposition; proper pair; isolated invariant set.