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# ON SEMICLASSICAL GROUND STATES FOR HAMILTONIAN ELLIPTIC SYSTEM WITH CRITICAL GROWTH

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Abstract. In this paper, we study the following Hamiltonian elliptic system with gradient term and critical growth:

$$\begin{cases} -\varepsilon^2\Delta\psi + \varepsilon b\cdot\nabla\psi + \psi = K(x)f(|\eta|)\varphi + W(x)|\eta|^{2^*-2}\varphi & \text{in } \mathbb{R}^N, \\ -\varepsilon^2\Delta\varphi - \varepsilon b\cdot\nabla\varphi + \varphi = K(x)f(|\eta|)\psi + W(x)|\eta|^{2^*-2}\psi & \text{in } \mathbb{R}^N, \end{cases}$$

where $\eta = (\psi,\varphi)\colon \mathbb{R}^N \to \mathbb{R}^2$, $K, W \in C(\mathbb{R}^N,\mathbb{R})$, $\varepsilon$ is a small positive parameter and $b$ is a constant vector. We require that the nonlinear potentials $K$ and $W$ have at least one global maximum. Combining this with other suitable assumptions on $f$, we prove the existence, exponential decay and concentration phenomena of semiclassical ground state solutions for all sufficiently small $\varepsilon > 0$.

## 1. Introduction and main results

We study the following Hamiltonian elliptic system with gradient term and critical growth:

$$(\mathcal{P}_\varepsilon) \qquad \begin{cases} -\varepsilon^2\Delta\psi + \varepsilon b\cdot\nabla\psi + \psi = K(x)f(|\eta|)\varphi + W(x)|\eta|^{2^*-2}\varphi & \text{in } \mathbb{R}^N, \\ -\varepsilon^2\Delta\varphi - \varepsilon b\cdot\nabla\varphi + \varphi = K(x)f(|\eta|)\psi + W(x)|\eta|^{2^*-2}\psi & \text{in } \mathbb{R}^N, \end{cases}$$

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where $\eta = (\psi,\varphi)\colon \mathbb{R}^N \to \mathbb{R}^2$, $N \geq 3$, $\varepsilon > 0$ is a small parameter, $2^* = 2N/(N-2)$ is the usual critical exponent and $b$ is a constant vector, and $f$ is a superlinear and subcritical nonlinearity. In this paper, we are concerned with the existence, exponential decay and concentration phenomenon of semiclassical ground state solutions of system $(\mathcal{P}_\varepsilon)$.

Systems $(\mathcal{P}_\varepsilon)$ or similar to $(\mathcal{P}_\varepsilon)$ were studied by a number of authors. But most of them focused on the case $b = 0$. For example, see [3]–[5], [9], [12], [20], [24], [26], [29], [32], [33], [35]–[38], [40], [41], [43] and the references therein. When $b \neq 0$ and $\varepsilon = 1$, there are not so many works on elliptic systems with the gradient term. Zhao and Ding [39] considered the following system:

$$(1.1)\qquad \begin{cases} -\Delta\psi + b(x)\cdot\nabla\psi + V(x)\psi = H_\varphi(x,\psi,\varphi) & \text{in } \mathbb{R}^N, \\ -\Delta\varphi - b(x)\cdot\nabla\varphi + V(x)\varphi = H_\psi(x,\psi,\varphi) & \text{in } \mathbb{R}^N, \end{cases}$$

where $b = (b_1,\ldots,b_N) \in C^1(\mathbb{R}^N,\mathbb{R}^N)$, $V \in C(\mathbb{R}^N,\mathbb{R})$ and $H \in C^1(\mathbb{R}^N\times\mathbb{R}^2,\mathbb{R})$. In this case, the appearance of the gradient term in this system brings some difficulties, and the variational framework for the case $b = 0$ cannot work any longer. Hence the authors first established suitable variational framework through the studying of the spectrum of operator, and obtained the multiplicity of solution for the non-periodic asymptotically quadratic case by applying the theorems of Bartsch and Ding [6]. Moreover, without the assumption that $H(x,\eta)$ is even in $\eta$, infinitely many geometrically distinct solutions for the periodic asymptotically quadratic case were obtained by using a reduction method. For the periodic superquadratic case, Zhang et al. [42] proved the existence of ground state solution for system (1.1). Recently, Yang et al. [34] considered the non-periodic superquadratic system

$$(1.2)\qquad \begin{cases} -\Delta\psi + b\cdot\nabla\psi + \psi = H_\varphi(x,\psi,\varphi) & \text{in } \mathbb{R}^N, \\ -\Delta\varphi - b\cdot\nabla\varphi + \varphi = H_\psi(x,\psi,\varphi) & \text{in } \mathbb{R}^N, \end{cases}$$

with a constant vector $b$. Since the problem is set in unbounded domain with non-periodic nonlinearities, the $(C)_c$-condition does not hold in general. To overcome the difficulty, they first considered certain limit problem related to system (1.2) which is autonomous, and constructed linking levels of the variational functional and proved the $(C)_c$-condition.

For small $\varepsilon > 0$ the solutions (standing waves) of $(\mathcal{P}_\varepsilon)$ are referred to as semiclassical states, which describe the transition from quantum mechanics to classical mechanics when the parameter $\varepsilon$ goes to zero, and possess an important physical interest. For such case, the asymptotic behavior of semiclassical states, such as concentration, exponential decay, etc., is a very interesting problem in mathematics and physics. To the best of our knowledge, there is only a few works concerning the existence and concentration phenomena of semiclassical states.