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CORRIGENDUM TO "THE SPLITTING LEMMAS FOR NONSMOOTH FUNCTIONALS ON HILBERT SPACES II. THE CASE AT INFINITY" (TOPOL. METHODS NONLINEAR ANAL. 44 (2014), 277–335)

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Abstract. We show how to correct errors in [1, § 4] caused by the incorrect inequality [1, (4.2)].

Here we only point out main corrected points and refer readers to [2, § 4] for a completely rewritten version of [1, § 4]. After removing the incorrect inequality [1, (4.2)] some corrections to the arguments in [1, § 4] should be made.

• The original (q_1^*) and (q_3^*) should be replaced by the following slightly stronger ones:

(q_1^*) There exist constants $c_1 > 0$, $r \in (0,1)$ and a function $E \in L^2(\Omega)$ such that $|q(x,t)| \le E(x) + c_1|t|^r$ for almost $x \in \Omega$ and for all $t \in \mathbb{R}$.

(q_3^*) For almost every $x \in \Omega$ the function $\mathbb{R} \ni t \mapsto q(x,t)$ is differentiable and $\Omega \times \mathbb{R} \ni (x,t) \mapsto q_t(x,t) := \frac{\partial q}{\partial t}(x,t)$ is a Carthéodory function. There exist $s \in (n/2, \infty)$, $\ell \in L^s(\Omega)$, and a bounded measurable $h\colon \mathbb{R} \to \mathbb{R}$ such that $h(t) \to \hbar \in \mathbb{R}$ as $|t| \to \infty$ and $|q_t(x,t)| \le \ell(x)h(t)$ for almost every $x \in \Omega$ and for all $t \in \mathbb{R}$.

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By the latter, $s \in (n/2, \infty)$, and so $s/(s-1) < n/(n-2)$ for $n > 2$. Set

$$\xi(s,n) = \begin{cases} \dfrac{s}{s-1} + \dfrac{n}{n-2} & \text{if } n > 2, \\ \dfrac{3s}{s-1} & \text{if } n = 2, \end{cases} \tag{0.1}$$

and

$$\eta(s,n) = \begin{cases} \dfrac{s}{2}\dfrac{2sn-2s-n}{s^2-s-n} & \text{if } n > 2, \\ \dfrac{3s}{s-1} & \text{if } n = 2. \end{cases} \tag{0.2}$$

Note that $H = H^1_0(\Omega) \hookrightarrow L^{\xi(s,n)}(\Omega)$. Let $c(s,n,\Omega) > 0$ be the best constant such that

$$\|u\|_{L^{\xi(s,n)}} \le c(s,n,\Omega)\|\nabla u\|_{L^2} = c(s,n,\Omega)\|u\|_H \quad \text{for all } u \in H. \tag{0.3}$$

• Two lines above Proposition 4.2 of [1] should be changed into:

Since $|q_t(x,t)| \le \ell(x)h(t)$ by (q^*_3), $1/s + 1/\eta(s,n) + 2/\xi(s,n) = 1$, $\eta(s,n) > 1$, and $2s/(s-1) < \xi(s,n) < 2n/(n-2)$ for $n > 2$, using the generalized Hölder inequality and Sobolev embedding theorem, we deduce

$$\begin{aligned} \left|\int_\Omega q_t(x,u(x))v(x)w(x)\,dx\right| &\le \int_\Omega |\ell(x)|\cdot|h(u(x))|\cdot|v(x)|\cdot|w(x)|\,dx \\ &\le \|\ell\|_{L^s}\|v\|_{L^{\xi(s,n)}}\|w\|_{L^{\xi(s,n)}}\biggl(\int_\Omega |h(u(x))|^{\eta(s,n)}\,dx\biggr)^{1/\eta(s,n)} \\ &\le (c(s,n,\Omega))^2\|\ell\|_{L^s}\|v\|_H\|w\|_H\biggl(\int_\Omega |h(u(x))|^{\eta(s,n)}\,dx\biggr)^{1/\eta(s,n)} \qquad (0.4) \\ &\le (c(s,n,\Omega))^2\|\ell\|_{L^s}\|v\|_H\|w\|_H|\Omega|^{1/\eta(s,n)}\sup h \qquad (0.5) \end{aligned}$$

for any $u, v, w \in H$. It follows that $B(u) \in L_s(H)$.

• (b) of [1, Proposition 4.2] should be replaced by

(b) Under the assumption (q^*_3), J is C^2 and $J''(u) := D(\nabla J)(u) = B(u)$ for all $u \in H$. Moreover, if $a = \lambda_m$ it holds with the constant $c(s,n,\Omega)$ in (0.4) that

$$\begin{aligned} \|g''(z+u)\|_{\mathcal{L}(H)} \le{}& (c(s,n,\Omega))^2\|\ell\|_{L^s}\|h\circ(z+u) - \hbar\|_{L^{\eta(s,n)}} \\ &+ (c(s,n,\Omega))^2|\Omega|^{1/\eta(s,n)}\|\ell\|_{L^s}\hbar \end{aligned} \tag{0.6}$$

for any $z \in H^0_\infty = \mathrm{Ker}(B(\infty))$ and $u \in H^\pm_\infty := (H^0_\infty)^\perp$.

• The last two lines on [1, 325] (or the equalities [1, (4.13)]) should be removed. And [1, Claim 4.4] should be replaced by:

Claim 4.4. *For given numbers $\rho > 0$ and $\varepsilon > 0$ there exists $R_0 > 0$ such that*

$$\|h(z+u) - \hbar\|_{L^{\eta(s,n)}} + \hbar|\Omega|^{1/\eta(s,n)} < \varepsilon + \hbar|\Omega|^{1/\eta(s,n)}$$