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MULTIPLICITY OF SOLUTIONS FOR p-LAPLACIAN TYPE ELLIPTIC PROBLEMS WITH ELECTROMAGNETIC FIELDS AND CRITICAL NONLINEARITY

Zhongyi Zhang

Abstract. We consider a class of p-Laplacian type elliptic problems with electromagnetic fields and critical nonlinearity in bounded domains. New results about the existence and multiplicity of solutions to these problems are obtained by using the concentration-compactness principle and variational method.

1. Introduction

In this paper we deal with the existence and multiplicity of solutions to the following p-Laplacian type elliptic problems with electromagnetic fields and critical nonlinearity:

$$\begin{cases} \left[g\left(\displaystyle\int_\Omega |\nabla_A u|^p\,dx\right)\right]\Delta_{p,A}u = \lambda h(x,|u|^p)|u|^{p-2}u + |u|^{p^*-2}u, & x \in \Omega, \\ u = 0, & x \in \partial\Omega, \end{cases} \tag{1.1}$$

where $\Delta_{p,A}u(x) := \operatorname{div}(|\nabla u + iA(x)u|^{p-2}(\nabla u + iA(x)u)$, here i is the imaginary unit, $\Omega \subset \mathbb{R}^N$ is an open bounded domain with smooth boundary and λ is a positive parameter, $p^* = Np/(N-p)$ is the critical exponent according to

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the Sobolev embedding. Functions $h\colon \overline{\Omega}\times\mathbb{R}\to\mathbb{R}$, $g\colon \mathbb{R}^+\to\mathbb{R}^+$ are continuous functions that satisfy the following conditions:

(G1) There exists $\alpha_0>0$ such that $g(t)\geq\alpha_0$ for all $t\geq 0$.
(G2) There exists σ such that $1<p/\sigma<p^*$ and $G(t)\geq\sigma g(t)t$ for all $t\geq 0$, where $G(t)=\int_0^t g(s)\,ds$.
(H1) $h(x,s)\in C(\Omega\times\mathbb{R},\mathbb{R})$, $h(x,-s)=-h(x,s)$ for all $s\in\mathbb{R}$.
(H2) $\lim\limits_{|s|\to\infty} h(x,s)/s^{(p^*-p)/p}=0$ uniformly for $x\in\Omega$.
(H3) $\lim\limits_{|s|\to 0^+} h(x,s)/s^{1/\sigma-1}=\infty$ uniformly for $x\in\Omega$.

There is a vast literature concerning the existence and multiplicity of solutions for (1.1) with no magnetic field, namely $A(x)\equiv 0$, $g(t)\equiv 1$ and $p=2$, starting from the celebrated paper by Brezis and Nirenberg [2]. For example, Li and Zou [28] obtained infinitely many solutions with odd nonlinearity. Chen and Li [7] established the existence of infinitely many solutions by using the minimax procedure. For more related results, we refer the interested readers to [3], [5], [14], [15], [17], [22], [34] and references therein.

On the one hand, for the special case of problem (1.1) with $A(x)\equiv 0$, $g(t)=at+b$ and $p=2$, equation (1.1) reduces to the following Dirichlet problem of Kirchhoff type:

$$\begin{cases} -\left(a+b\displaystyle\int_\Omega|\nabla u|^2\,dx\right)\Delta u=f(x,u), & x\in\Omega,\\ u|_{\partial\Omega}=0, \end{cases}\tag{1.2}$$

where $\Omega\subset\mathbb{R}^N$, problem (1.2) is a generalization of a model introduced by Kirchhoff [23]. More precisely, Kirchhoff proposed a model given by the equation

$$\rho\frac{\partial^2 u}{\partial t^2}-\left(\frac{\rho_0}{h}+\frac{E}{2L}\int_0^L\left|\frac{\partial u}{\partial x}\right|^2\,dx\right)\frac{\partial^2 u}{\partial x^2}=0,\tag{1.3}$$

where ρ,ρ_0,h,E,L are constants, which extends the classical D'Alembert's wave equation, by considering the effects of changes in the length of strings during vibrations. Equation (1.2) is related to the stationary analogue of problem (1.3). Problem (1.2) received much attention only after Lions [26] proposed an abstract framework to the problem. Some important and interesting results can be found in, for example, [11], [10], [18], [20], [21], [24], [30], [32], [39]. We note that the results dealing with problem (1.2) with critical nonlinearity are relatively scarce. For the case $p\neq 2$, by means of a direct variational method, the authors proved the existence and multiplicity of solutions to a class of p-Kirchhoff-type problem with Dirichlet boundary data [12]. In [29], the author showed the existence of infinite solutions to the p-Kirchhoff type quasilinear elliptic equation. But they did not give any further information on the sequence of solutions.