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THE BANACH–MAZUR DISTANCE BETWEEN $C(\Delta)$ AND $C_0(\Delta)$ EQUALS 2

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Dedicated to the memory of Professor Kazimierz Goebel

Abstract. Let $C(\Delta)$ denote the Banach space of all continuous real-valued functions on the Cantor set Δ and $C_0(\Delta) = \{f \in C(\Delta) : f(1) = 0\}$. From the 1966 theorem of Cambern, it is well-known that the Banach–Mazur distance $d(C(\Delta), C_0(\Delta)) \geq 2$. We prove that, in fact, $d(C(\Delta), C_0(\Delta)) = 2$. As a consequence, we answer a question left open in the 2012 paper of Candido and Galego.

1. Introduction

For a locally compact Hausdorff space K, $C_0(K)$ denotes the Banach space of all continuous real-valued functions on K which vanish at infinity, endowed with the supremum norm; it is said that a continuous function $f\colon K \to \mathbb{R}$ vanishes at infinity if the set $\{x \in K : |f(x)| \geq \varepsilon\}$ is compact for every $\varepsilon > 0$. If K is compact, then $C_0(K)$ consists of all continuous real-valued functions on K and this space will be denoted by $C(K)$. In the abstract we denoted the space $\{f \in C(\Delta) : f(1) = 0\}$ by $C_0(\Delta)$. Obviously, this notation does not fit the standard meaning of $C_0(K)$ for a locally compact Hausdorff space K because Δ is compact but $C_0(\Delta)$ is not isometric to $C(\Delta)$. However, in this particular case, when $K = \Delta$, we prefer to use the notation $C_0(\Delta)$ to represent the space $\{f \in C(\Delta) : f(1) = 0\}$ because it is more natural for our purposes.

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