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A BOREL LINEAR SUBSPACE OF $\mathbb{R}^\omega$ THAT CANNOT BE COVERED BY COUNTABLY MANY CLOSED HAAR-MEAGER SETS

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Dedicated to the memory of Professor Kazimierz Goebel

Abstract. We prove that the countable product of lines contains a Haar-null Haar-meager Borel linear subspace L that cannot be covered by countably many closed Haar-meager sets. This example is applied to studying the interplay between various classes of "large" sets and Kuczma–Ger classes in the topological vector spaces $\mathbb{R}^n$ for $n \leq \omega$.

1. Introduction

By the classical Steinhaus Theorem [15], for any Borel subset A of positive Lebesgue measure on the real line, the difference $A - A = \{x - y : x, y \in A\}$ is a neighbourhood of zero. In [16] Weil extended this result of Steinhaus to all locally compact Polish Abelian groups proving that for any Borel subset A of positive Haar measure in such a group X, the set $A-A$ is a neighbourhood of the neutral element of X. This result implies that any nonopen Borel subgroup of a locally compact Abelian Polish group X belongs to the σ-ideal $\mathcal{N}_X$ of subsets of Haar measure zero in X.

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Key words and phrases. Additive function; mid-convex function; continuity; Haar-null set; Haar-meager set; null-finite set; Haar-thin set; Polish Abelian group; Ger–Kuczma classes.

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