

## CONLEY INDEX THEORY FOR GUTIERREZ–SOTOMAYOR FLOWS ON SINGULAR 3-MANIFOLDS

KETTY A. DE REZENDE — NIVALDO G. GRULHA JR.  
DAHISY V. DE S. LIMA — MURILO A.J. ZIGART

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**ABSTRACT.** This paper is a continuation of the investigation done in dimension two, this time for the Gutierrez–Sotomayor vector fields on singular 3-manifolds. The singularities of Gutierrez–Sotomayor flows (GS flows, for short) in this setting are the 3-dimensional counterparts of cones, cross-caps, double and triple crossing points. First, we prove the existence of a Lyapunov function in a neighborhood of a given singularity of a GS flow, i.e. a GS singularity. In these neighbourhoods, index pairs are defined and allow a direct computation of the Conley indices for the different types of GS singularities. The Conley indices are used to prove local necessary conditions on the number of connected boundary components of an isolating block for a GS singularity as well as their Euler characteristic. Lyapunov semi-graphs are introduced as a tool to record this topological and dynamical information. Lastly, we construct isolating blocks so as to prove the sufficiency of the connectivity bounds on the boundaries of isolating blocks given by the Lyapunov semi-graphs.

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*Key words and phrases.* Conley index; Lyapunov function; Lyapunov graph; cobordisms; handle theory.

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